Space-Efficient Data Structures for Strings and Sequences

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Joint works with

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Why string data?

Ubiquitous and massive data:

- Digital libraries and product catalogues
- Electronic white and yellow pages
- Specialized information sources (e.g., genome or patents)
- Web pages repositories
- XML format for structured data
- Newsgroups
- ...

What is a String Dictionary?

- Persistent and space-efficient data structure.
- Quickly answer to lots of seach queries.
- Accessing a small portion of the string collection.

- Basic building block of any IR system
  (along with modeling, ranking, query languages, security and access control)

  For example...

Text indexing (TI): **world-level** vs full-text

"This rare goods [the information]

will be prepared under malleable or eatable form,

will be delivered to more and more customers;

it will be sold, exported, duplicated and reproduced

everywhere…"

Paul Valéry, Poet, 1871-1945

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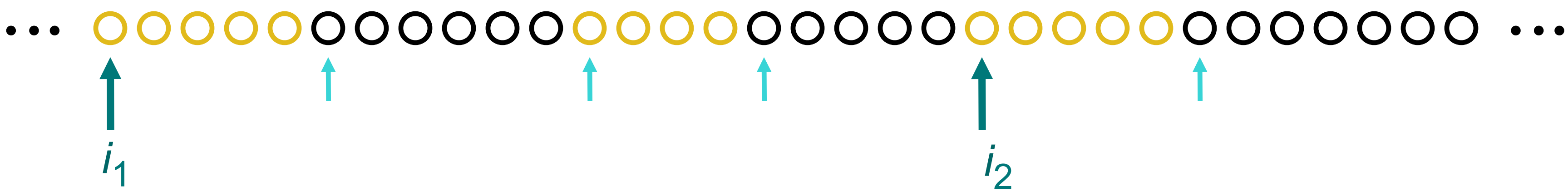
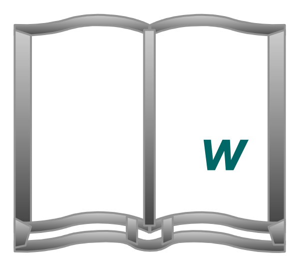
Word-level TI: Inverted lists/files



- Split the text into words

- Collect all distinct words in a string dictionary:
  for each word *w*, store the
  (inverted or position) list
  of its locations in the text



i_1, i_2, …

- Support Boolean/ranked queries

Search “will” + “and”

“This rare goods [the information]

will be prepared under malleable or eatable form,

will be delivered to more and more customers;

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everywhere, and …”

Paul Valéry, Poet, 1871-1945

Basic property of full-text indexing: **Prefix searching** in a string dictionary

Pattern P occurs at position *i* of text T
iff
P is prefix of a suffix of T

`mississippi`

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Example of string dictionaries
supporting prefix search,
you can make succinct
using the techniques seen in previous talks...

Trie T for a set S of strings

1. S empty → T empty
2. $S = S_A \cup S_B \ldots \cup S_Z$ (partition):

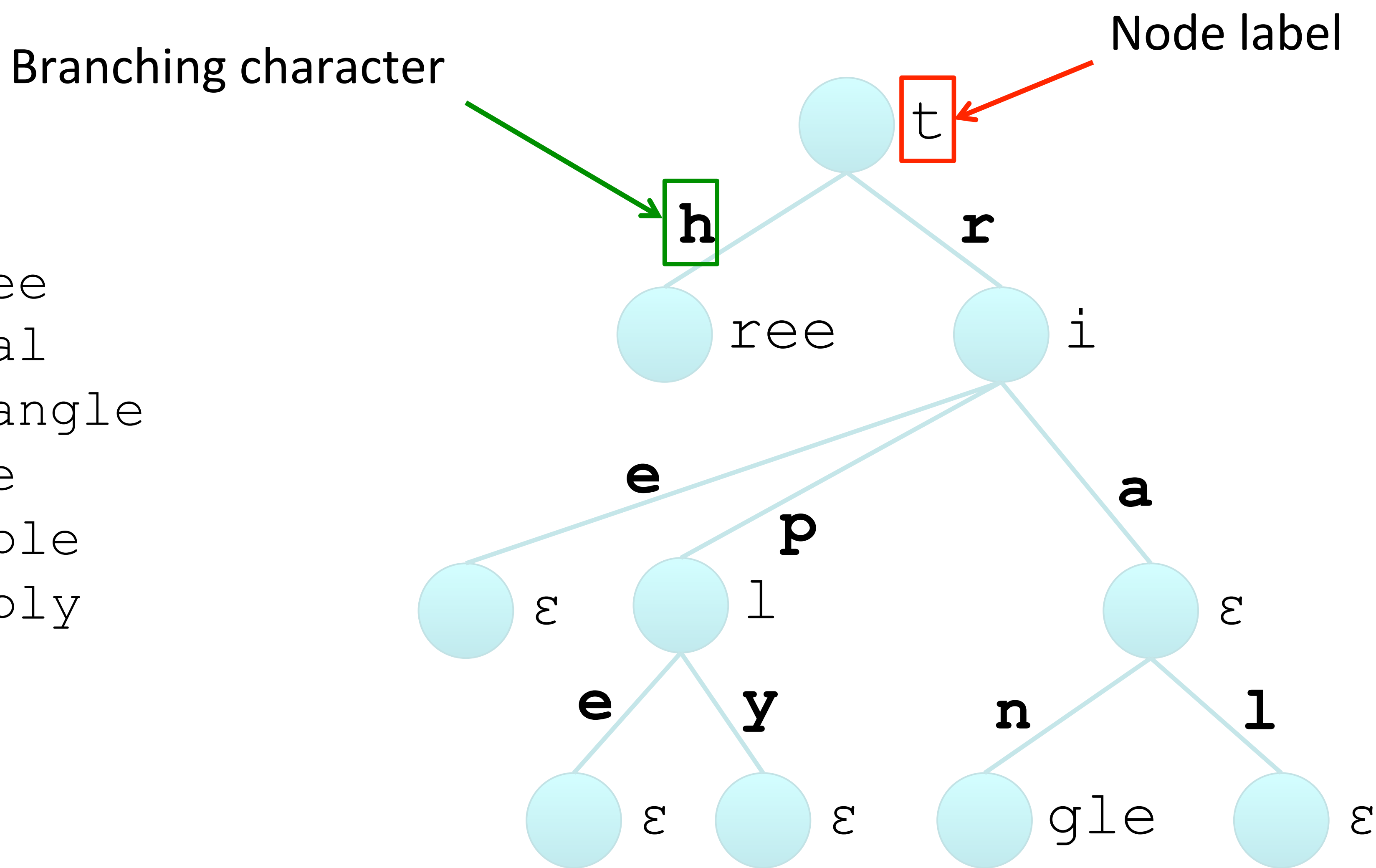
   T is the root with children

   $T_A, T_B, \ldots, T_Z$

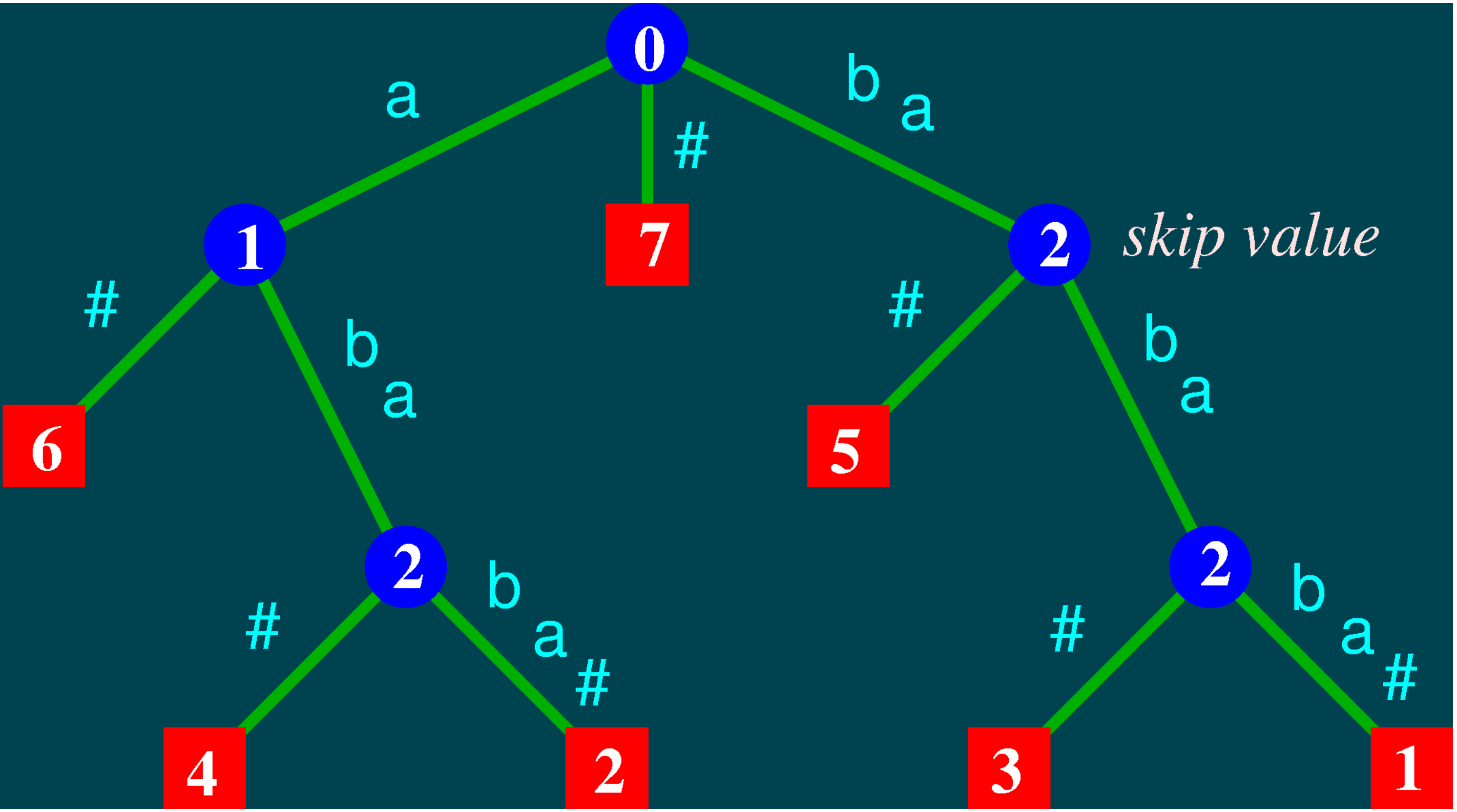
   with T_x = trie(S_x) with initial *x* removed

Compacted tries

three
trial
triangle
trie
triple
triply

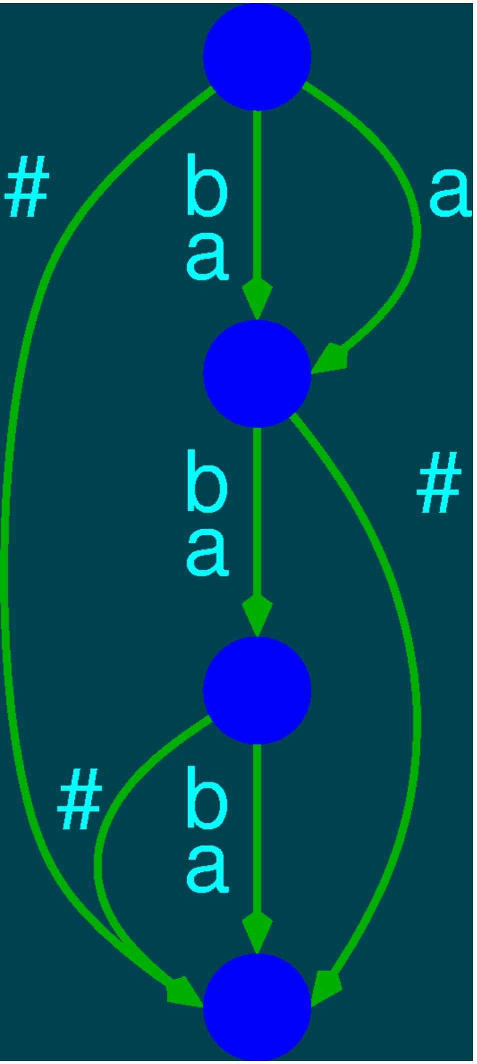


Full text index: Suffix tree



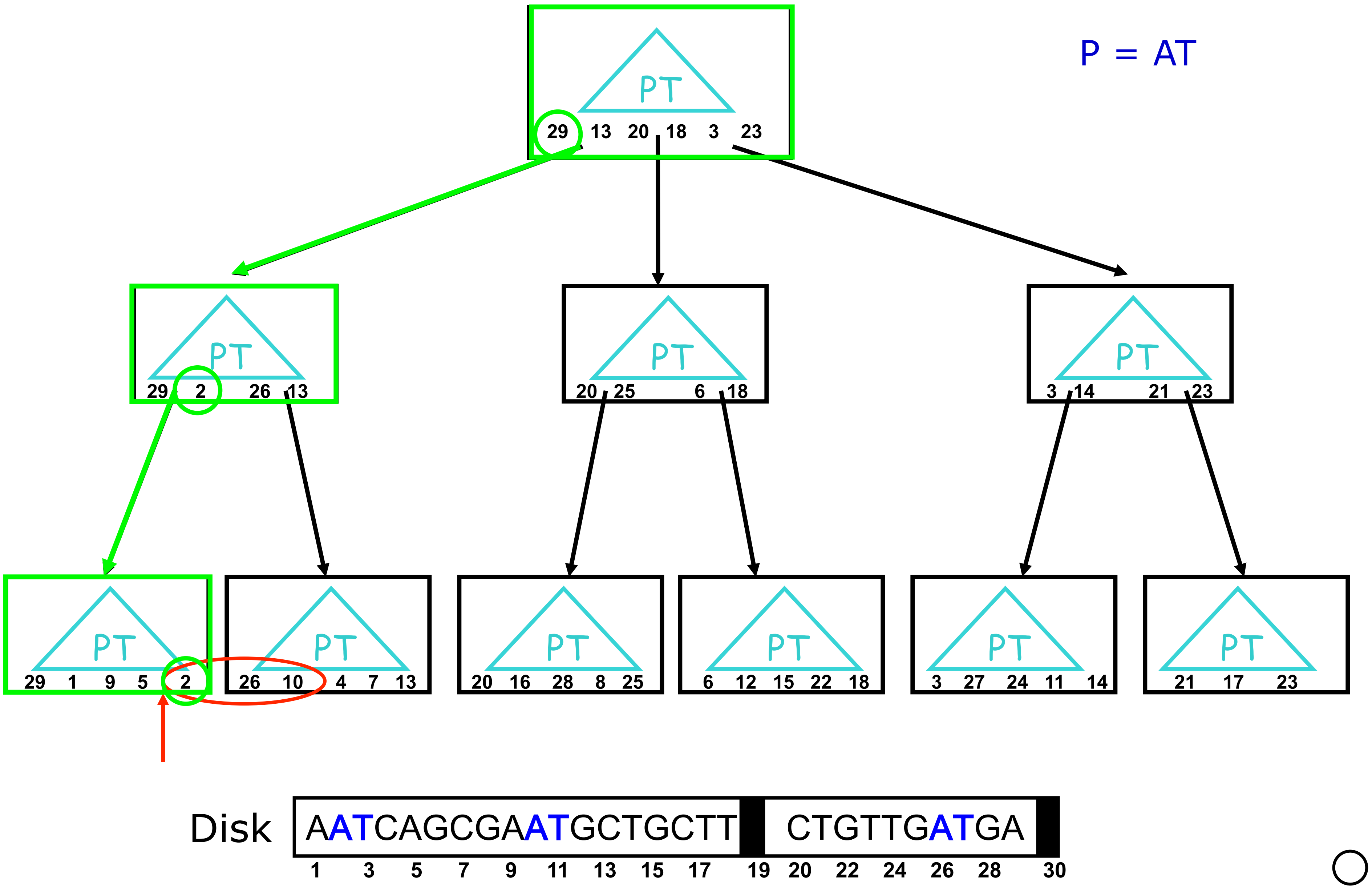
- Compact trie / Patricia storing the suffixes of bababa#
- Space is roughly 16*n* bytes [Manber,Myers]
- Online construction in linear time [Ukkonen]

Full text index: Automaton and DAWG



- Collapse isomorphic nodes to reduce their number
- Average size is about 1.3 nodes per character
- Nodes are of larger size
- Number of bytes does not reduce much

String B-trees



SA suffix array: `mississippi`

1	mississippi
2	ississippi
3	ssissippi
4	sissippi
5	issippi
6	ssippi
7	sippi
8	ippi
9	ppi
10	pi
11	i

11	i
8	ippi
5	issippi
2	ississippi
1	mississippi
10	pi
9	ppi
7	sippi
4	sissippi
6	ssippi
3	ssissippi

Let us focus on suffix arrays...

text T = S E N S E L E S S N E S S _

1	2	3	4	5	6	7	8	9	10	11	12	13	14
S	E	N	S	E	L	E	S	S	N	E	S	S	_

i	SA[i]	T[SA[i] ... n]
1	14	_
2	5	E L E S S N E S S _
3	2	E N S E L E S S N E S S _
4	11	E S S _
5	7	E S S N E S S _
6	6	L E S S N E S S _
7	10	N E S S _
8	3	N S E L E S S N E S S _
9	13	S _
10	4	S E L E S S N E S S _
11	1	S E N S E L E S S N E S S _
12	9	S N E S S _
13	12	S S _
14	8	S S N E S S _

Storing the SA permutation...

i	SA[i]	Φ(i)
1	14	0/11
2	5	6
3	2	8
4	11	13
5	7	14
6	6	5
7	10	4
8	3	10
9	13	1
10	4	2
11	1	3
12	9	7
13	12	9
14	8	12

"suffix link" for SA:
Φ **function** [G.&Vitter '00]

Φ(i) = k iff SA[k] = SA[i] + 1

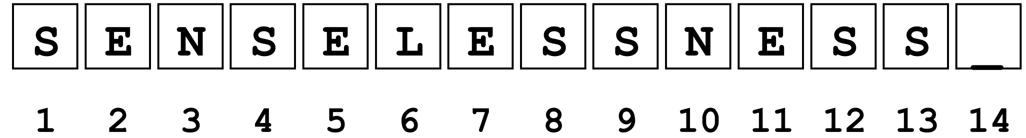
rank of the next suffix in T

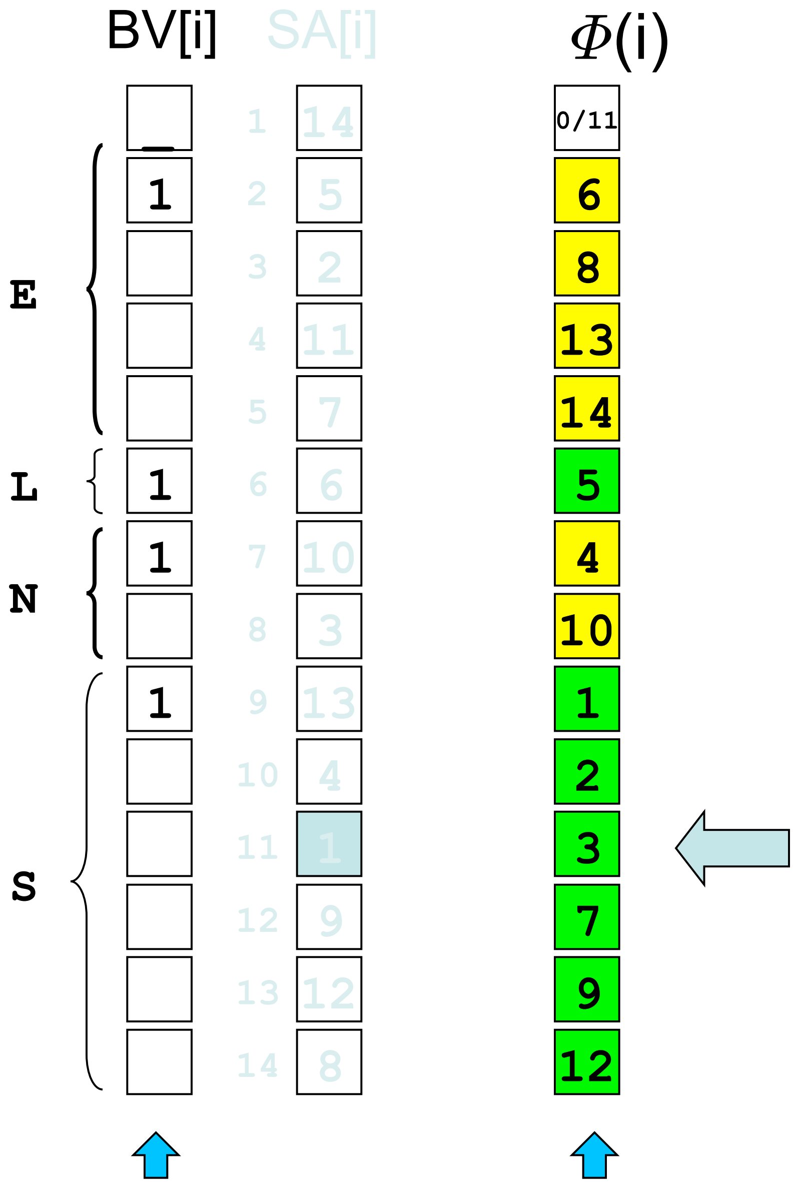
Φ(i)=0 when SA[i]=n or
Φ(i)=z where SA[z]=1

S	E	N	S	E	L	E	S	S	N	E	S	S	_
1	2	3	4	5	6	7	8	9	10	11	12	13	14

	BV[i]		SA[i]	T[SA[i]]	Φ(i)
	_	1	14	_	0/11
E	1	2	5	E	6
		3	2	E	8
		4	11	E	13
		5	7	E	14
L	1	6	6	L	5
N	1	7	10	N	4
		8	3	N	10
S	1	9	13	S	1
		10	4	S	2
		11	1	S	3
		12	9	S	7
		13	12	S	9
		14	8	S	12

Using Φ and a bitvector BV we can avoid storing SA





Using Φ and a bitvector BV we can avoid storing SA

S	E	N	S	E	L	E	S	S	N	E	S	S	_
1	2	3	4	5	6	7	8	9	10	11	12	13	14

	BV[i]	i	SA[i]	Φ(i)
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E	1	2	5	6
		3	2	8
		4	11	13
		5	7	14
L	1	6	6	5
N	1	7	10	4
		8	3	10
S	1	9	13	1
		10	4	2
		11	1	3
		12	9	7
		13	12	9
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S	E	N	S	E	L	E	S	S	N	E	S	S	_
1	2	3	4	5	6	7	8	9	10	11	12	13	14

	BV[i]		SA[i]	Φ(i)
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E		3	2	8
		4	11	13
		5	7	14
L	1	6	6	5
N	1	7	10	4
		8	3	10
	1	9	13	1
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		13	12	9
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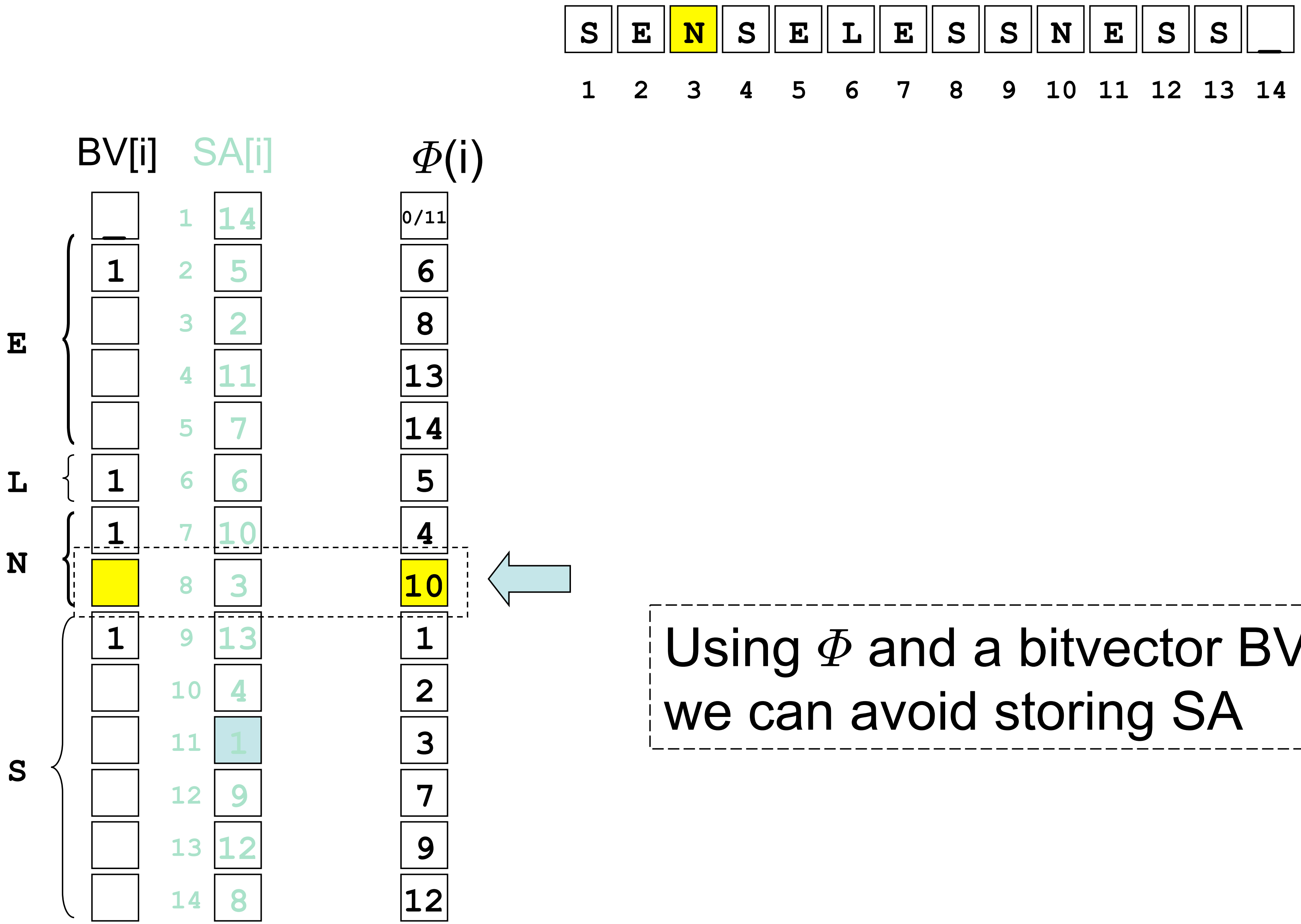

Using Φ and a bitvector BV we can avoid storing SA

S	E	N	S	E	L	E	S	S	N	E	S	S	_
1	2	3	4	5	6	7	8	9	10	11	12	13	14

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		4	11	13
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L	1	6	6	5
N	1	7	10	4
		8	3	10
S	1	9	13	1
		10	4	2
		11	1	3
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		13	12	9
		14	8	12

Using Φ and a bitvector BV we can avoid storing SA

1	2	3	4	5	6	7	8	9	10	11	12	13	14
S	E	N	S	E	L	E	S	S	N	E	S	S	_

	BV[i]	i	SA[i]	Φ(i)
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E		3	2	8
E		4	11	13
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L	1	6	6	5
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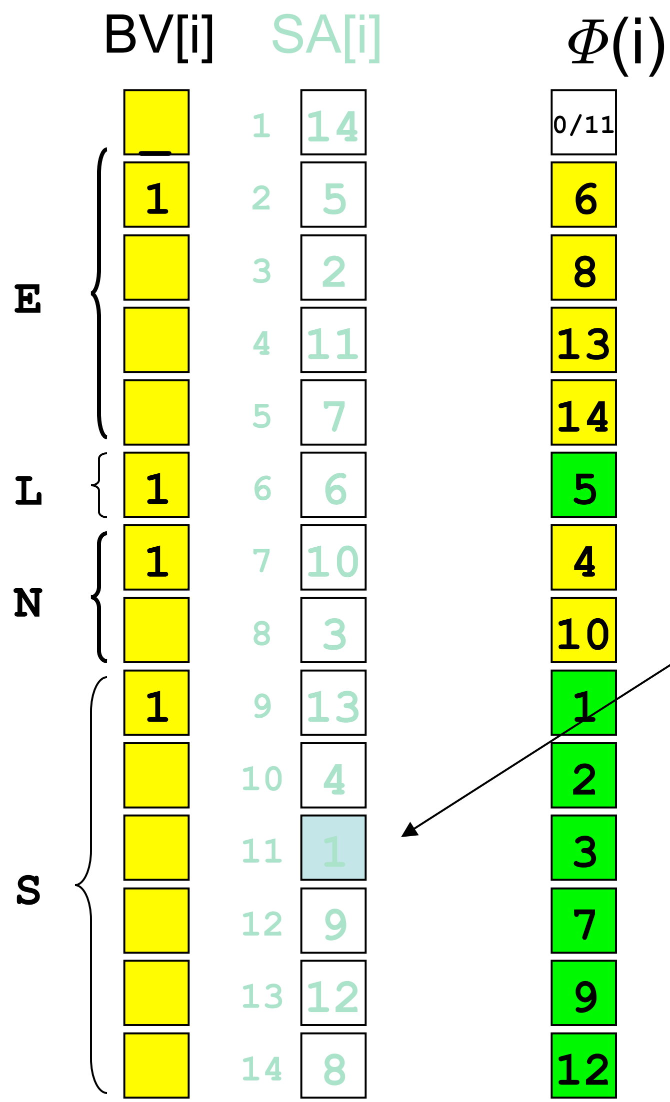
S	1	9	13	1
S		10	4	2
S		11	1	3
S		12	9	7
S		13	12	9
S		14	8	12

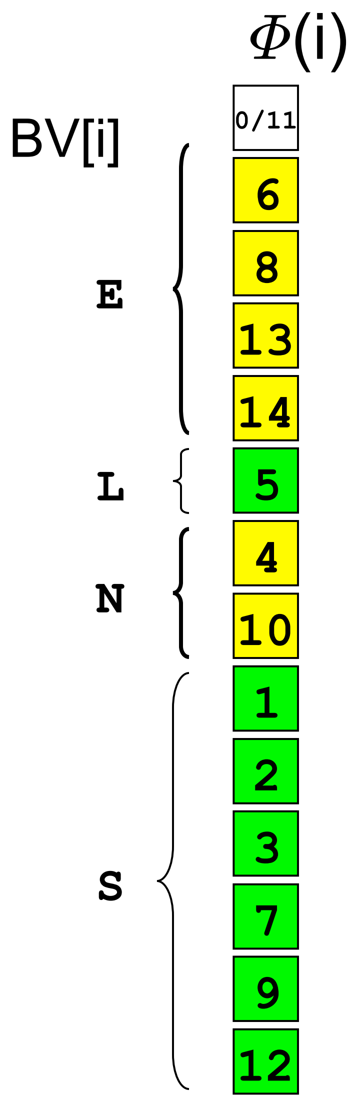
Using Φ and a bitvector BV we can avoid storing SA

S	E	N	S	E	L	E	S	S	N	E	S	S	_
1	2	3	4	5	6	7	8	9	10	11	12	13	14

Replacing SA by Φ and BV



- Record which SA[j] = 1
- T[i] = c iff $\Phi^{(i)}$(j) $\in$ Interval$_c$



This is what we need to store:

- bitvector BV
- Φ function

Also Φ is represented by bitvectors!

Exploit Φ's properties

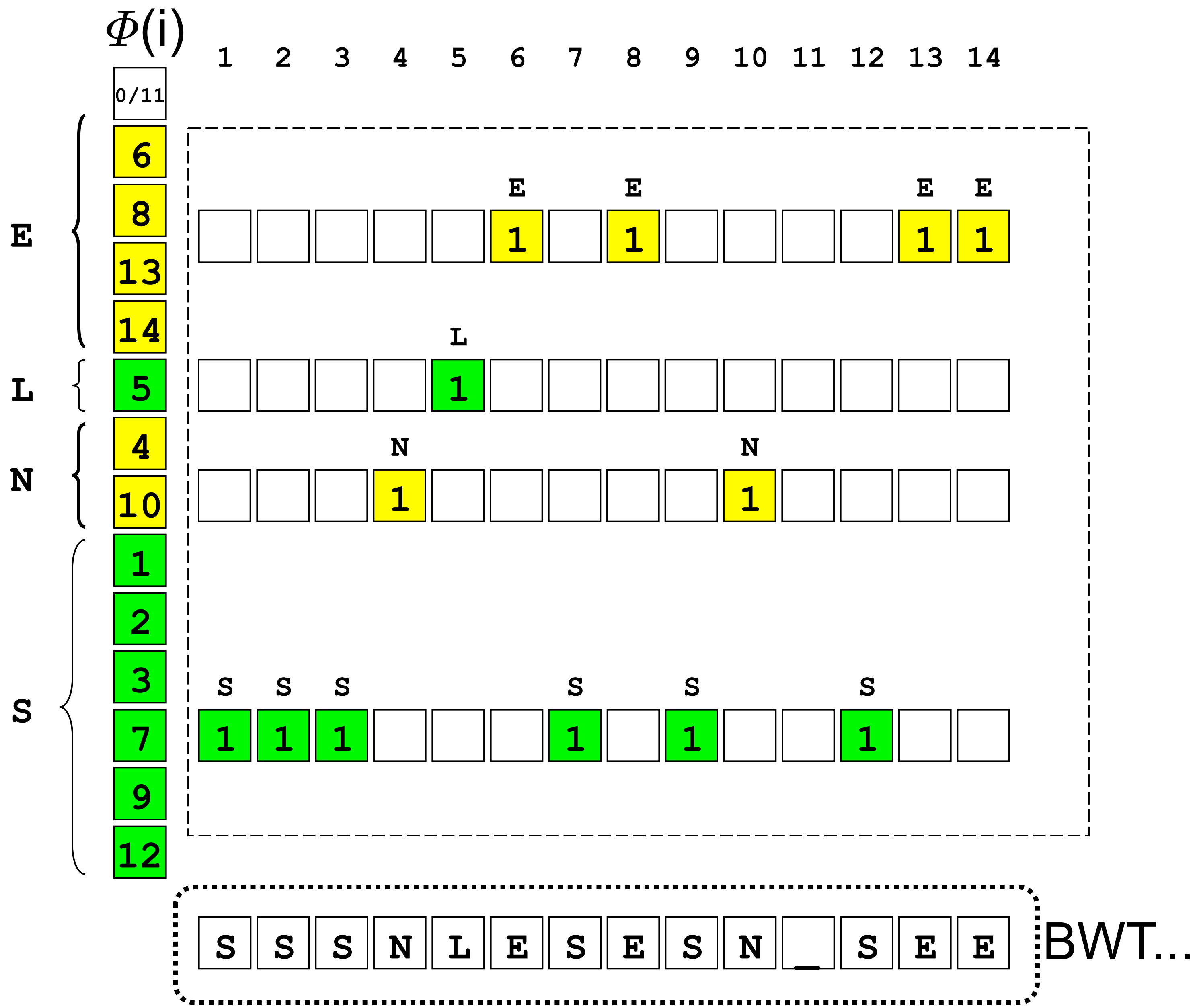
- Observation made on SA perms:

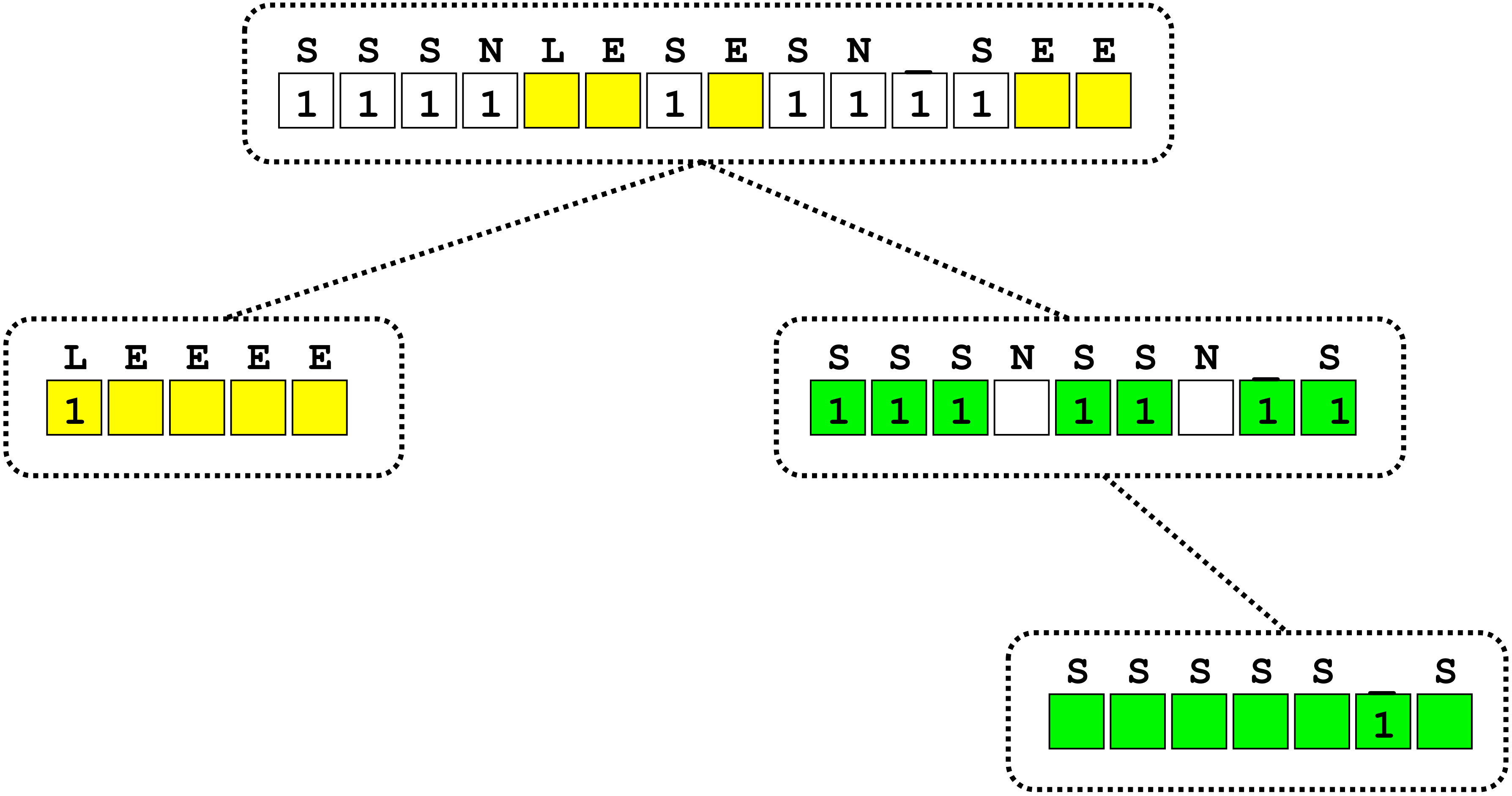
  T[SA[i]] = T[SA[i+1]] $\Rightarrow \Phi$(i) < Φ(i+1)

- Monotonicity: a maximal run of equal symbols

  T[SA[i]] = ⋯ = T[SA[j]] $\Rightarrow$ 1 $\leq \Phi$(i) < ⋯ < Φ(j) $\leq$ n

- At most σ runs, called Σ lists


Φ(i)
1 2 3 4 5 6 7 8 9 10 11 12 13 14
0/11
6
8
13
14
5
4
10
1
2
3
7
9
12
E
L
N
S
E E E E
1 1 1 1
L
1
N N
1 1
S S S S S S
1 1 1 1 1 1
S S S N L E S E S N _ S E E
BWT...



Wavelet tree on BWT

Entering compression stuff...

A first simple approach

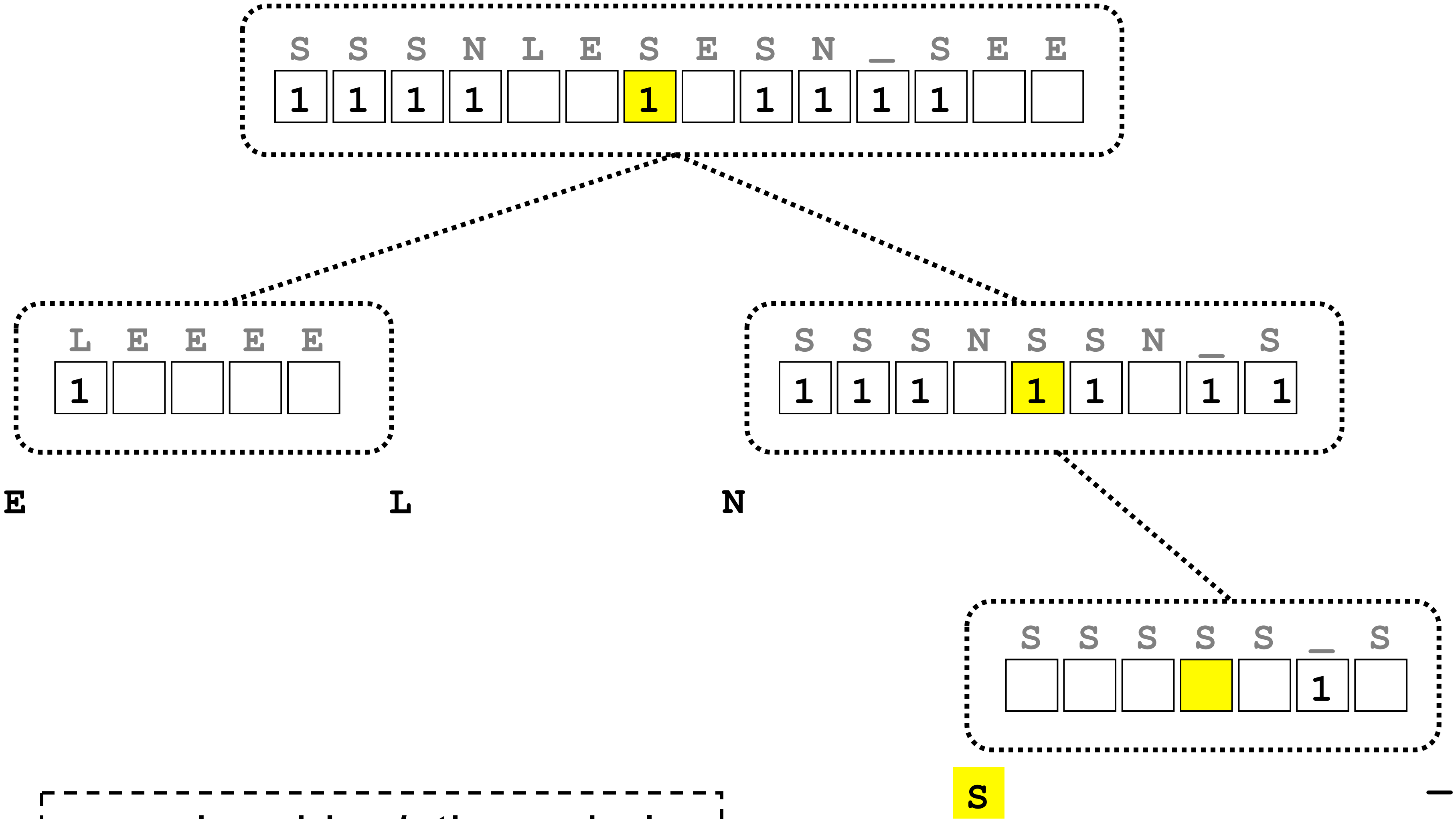
- Encode bitvector BV in B(σ,n) + o(n) bits
- Encode Φ function in B(n,nσ) + o(n log σ) bits

Using Wavelet Trees, compressed representation in

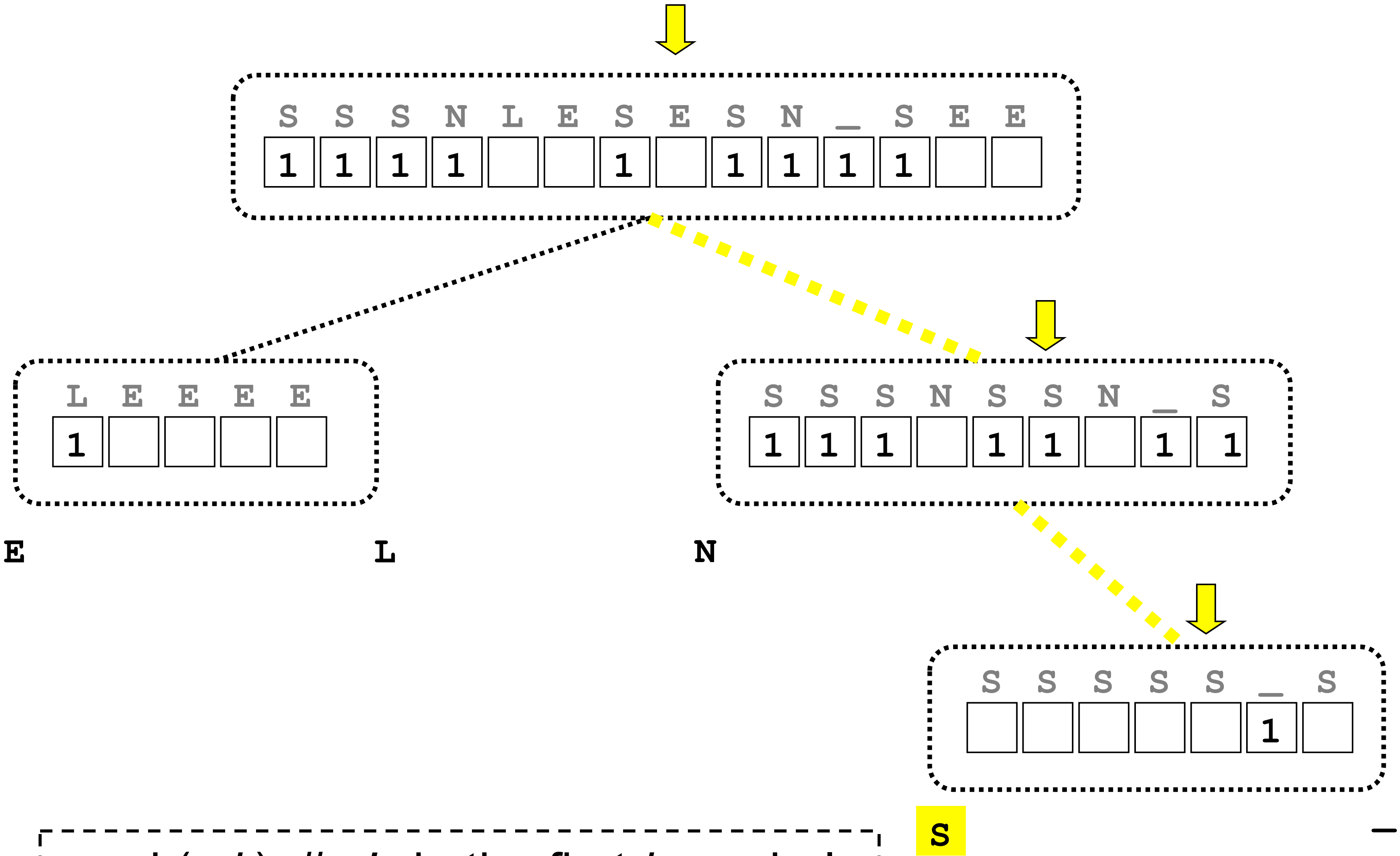
$\boxed{\text{n log } \sigma \text{ + lower order bits}}$

becomes high-order entropy **n H_k** by a refined analysis

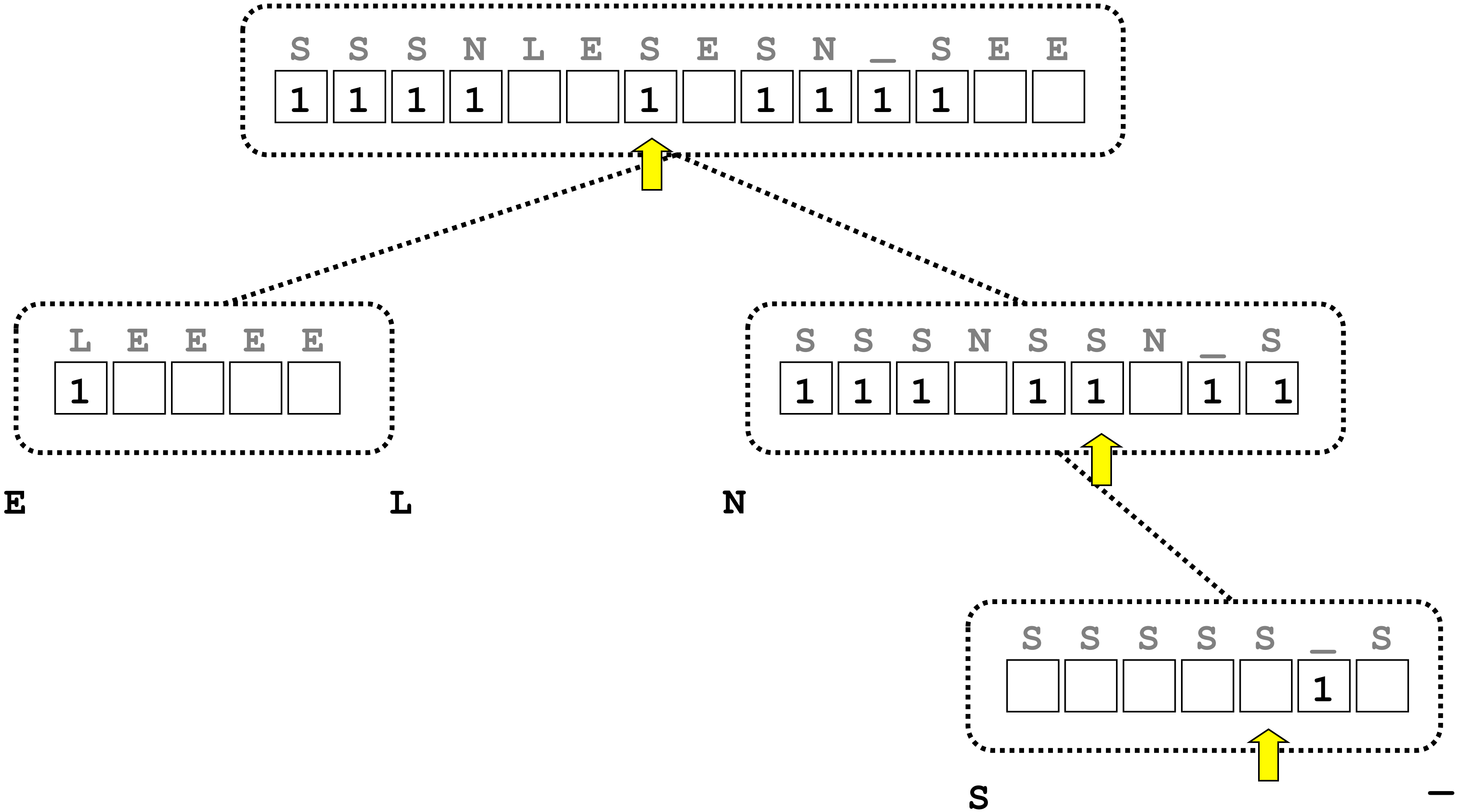
Let us focus on wavelet trees...



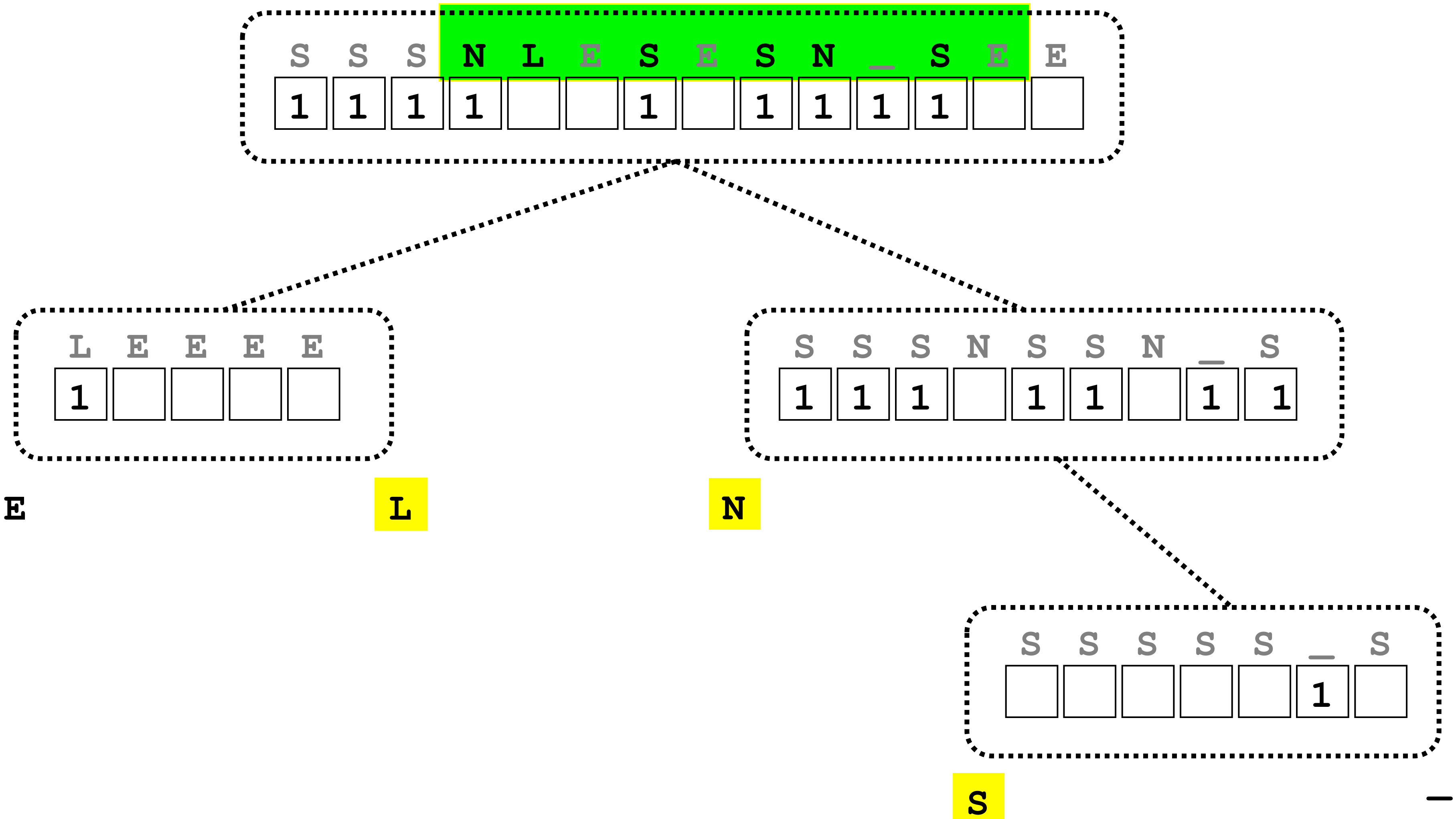
membership: *k*-th symbol



rank(c,k): # c's in the first k symbols



select(c,k): position of the k-th symbol c



2D-range($c..d$, $j..k$): like in Comp.Geom.

Wavelet tree represents…

- a **sequence of symbols** as we saw
- a **reordering** stored in the root of the multiset given by the elements in the leaves
- a **set of grid points**, where the x-coordinates are the positions and and the y-coordinates are the values

A list of applications [Navarro ’12]

- entropy encoding with direct access
- range queries
- positional inverted indexes
- graph representation
- permutations
- numeric sequences
- document retrieval
- binary relations
- data mining

… not always the best bounds, but easy to get them

Some theory

Quick Analysis of Space Occupancy

- B^j = j-th raw bit string in preorder traversal of the wavelet tree

- Fact 1. $\sum_j |B^j|\, H_0(B^j) = |string| \times H_0(string)$

- Fact 2 [Makinen-Navarro]. Wavelet on BWT(string)

$$\sum_j |B^j|\, H_0(B^j) = |string| \times H_k(string)$$

(order-k empirical entropy)

The rationale behind wavelet trees...

FOCUS on bit string $B \equiv B^j$

Fully indexable dictionaries (FID)

[Brodnik, Munro ‘99, Raman et al ’02]

- Bitvector V with n **1**s and m-n **0**s:
  - $rank_1(i)$ = #**1**s in V[1...i]
  - $select_1(j)$ = position in V of the jth **1**
  - same operations for **0**s
- Can also solve the precedessor problem

- space: B(n,m) + o(m) ~ n H_0 + o(m) bits
- time: O(1)

$$B(n,m) = \left\lceil \log \binom{m}{n} \right\rceil$$

Run Length Encoding (RLE) of a Bit String

- Bit string $B = B[1 \ldots n] = b_1^{l_1} b_2^{l_2} \ldots b_m^{l_m}$
  - $b_i \neq b_j$, if $i \neq j$
  - $l_i > 0$

- Prefix-free positive integer coding [Elias' 75]
  - $|\gamma(x)| = 2\lfloor \log x \rfloor + 1$ bits
  - $|\delta(x)| = \lfloor \log x \rfloor + 2\lfloor \log (\lfloor \log x \rfloor + 1) \rfloor + 1$ bits

$$\left.\begin{array}{l} \text{• RLE-}\gamma \text{ of B:} \quad b_1\, \gamma(l_1)\, \gamma(l_2) \ldots \gamma(l_m) \\ \text{• RLE-}\delta \text{ of B:} \quad b_1\, \delta(l_1)\, \delta(l_2) \ldots \delta(l_m) \end{array}\right\} \text{Uniquely decodable}$$

RLE of a Wavelet Tree

- T: an arbitrary text
  - Drawn from an alphabet of size σ
- $T_{rle,\gamma}$ = wavelet tree of T with each bit array RLE-γ encoded
- $T_{rle,\delta}$ = wavelet tree of T with each bit array RLE-δ encoded

Size of the RLE of a Wavelet Tree

- Total #bits in all the RLE encoded bit arrays

- $|B_{rle,\gamma}| \leq \mathbf{2}\ nH_0(B) + 2 \log n + 2$

  $|T_{rle,\gamma}| \leq \mathbf{2}\ nH_0(T)+(2\log n+2)(\sigma-1)$

- $|B_{rle,\delta}| \leq \mathbf{3}\ nH_0(B) + 2\log(\log n +1) + \log n + 2$

  $|T_{rle,\delta}| \leq \mathbf{3}\ nH_0(T)+(2\log(\log n + 1)+\log n + 2)(\sigma - 1)$

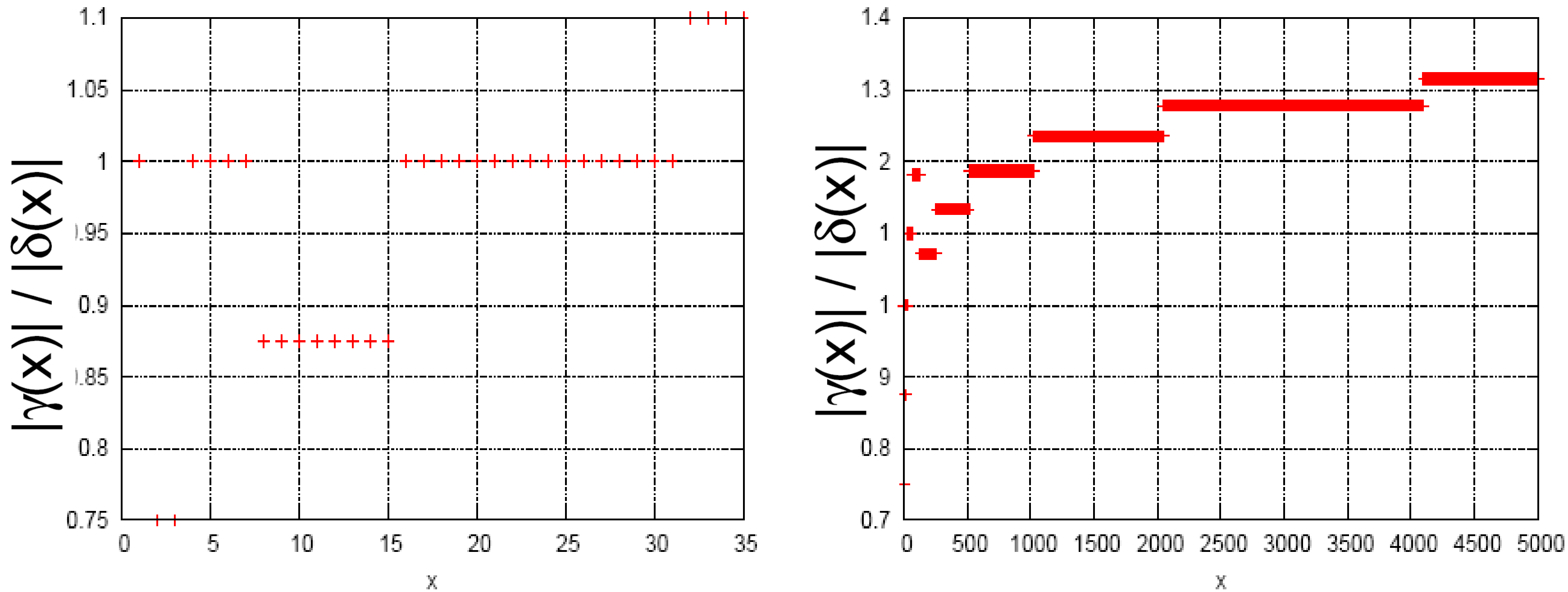
RLE-δ is less space efficient for an arbitrary text !!

Why is RLE-δ Less Space Efficient ?

- Although:
  - $\gamma(x)$ uses $\mathbf{2}\lfloor \log x \rfloor + 1$ bits
  - $\delta(x)$ uses $\lfloor \log x \rfloor + 2\lfloor \log (\lfloor \log x \rfloor + 1) \rfloor + 1$ bits

- Actually:
  $\delta(x)$ uses more bits if x is small, which is often the case in the wavelet tree bit arrays.
  [well-known fact]

Why is RLE-δ Less Space Efficient ? (Cont’d)



(a) Over a narrow range of x (b) Over a wide range of x

$|\delta(x)| < |\gamma(x)|$ for $x > 31$; $|\delta(x)| = |\gamma(x)|$ for $x \in \{1, 4\text{–}7, 16\text{–}31\}$; $|\delta(x)| > |\gamma(x)|$ for $x \in \{2\text{–}3, 8\text{–}15\}$.

A comparison of |γ(x)| and |δ(x)|

Can RLE-δ become provably better ?

- **Yes, when the data is highly skewed.**
- **For a bit string B**
  - If $H_0(B) = o(1)$:

    $|B_{rle,\delta}| \leq \mathbf{nH_0(B)} + o(nH_0(B)) + 2\log(\log n+1) + \log n + 2$ (optimal)

    (while $|B_{rle,\gamma}| \leq \mathbf{2nH_0(B)} + 2 \log n + 2$)

- **For an arbitrary text T from alphabet of size σ**
  - If $\sigma = O(1)$ and $H_0(T) = o(\log \sigma)$: (optimal)

    $|T_{rle,\delta}| \leq nH_0(T) + o(nH_0(T)) + (2\log(\log n + 1) + \log n + 2)(\sigma - 1)$

    (while $|T_{rle,\gamma}| \leq 2nH_0(T) + (2\log n + 2)(\sigma - 1)$)

Experiments

Software tool for wavelet trees

3 tree shapes (balanced, Huffman, Hu-Tucker)

X

11 coding schemes for bit strings B^i

- Can experiment 33 possible incarnations...
- URL: http://penguin.ewu.edu/~bojianxu/publications

Experimental results

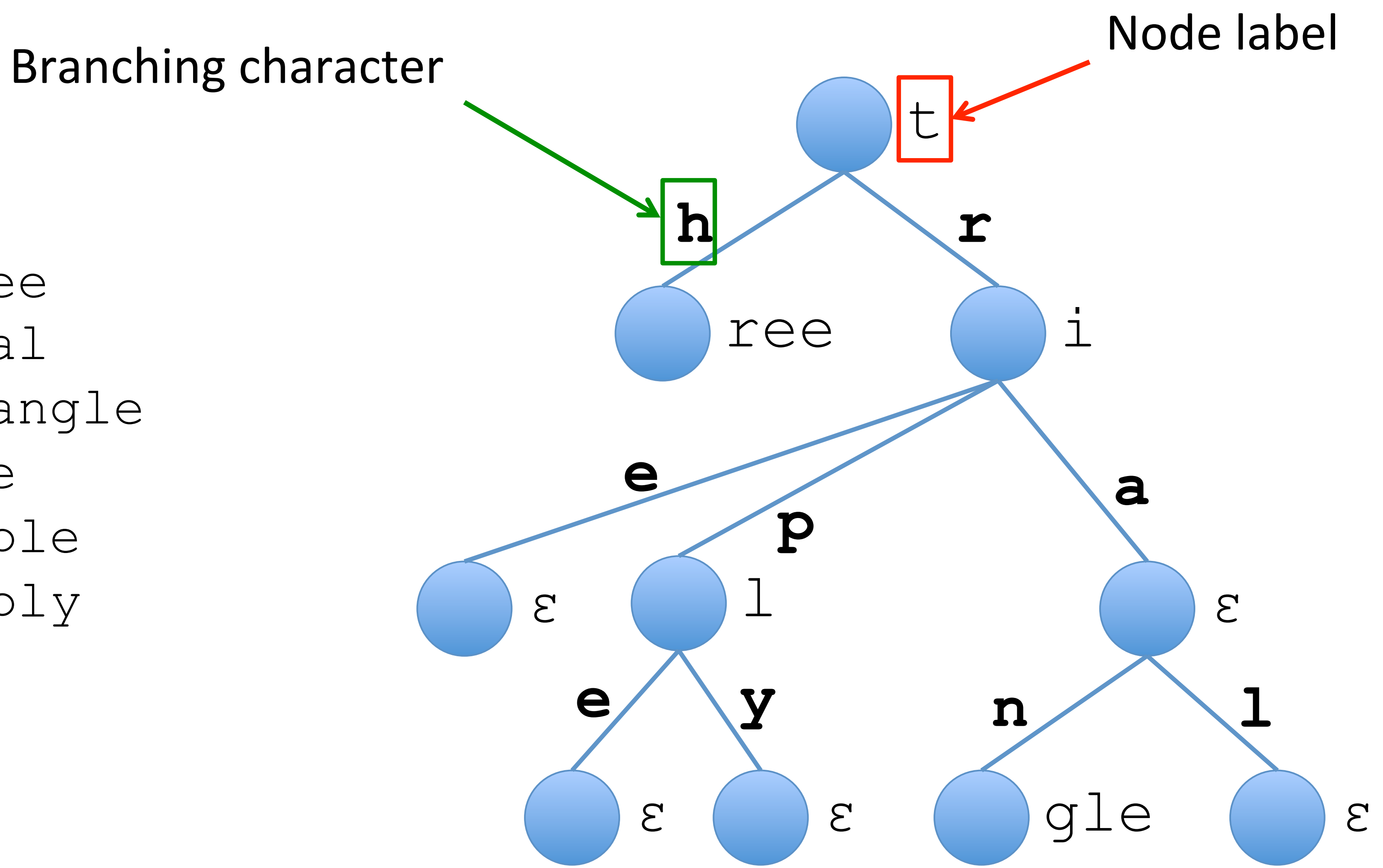
1. RLE+γ is good for low-entropy text
2. Hu-Tucker shape as good as Hufmann shape, except for low-entropy data
3. Queries are faster in Hu-Tucker shape

Part II

Fast Compressed Tries through Path Decompositions

Compacted tries

```
three
trial
triangle
trie
triple
triply
```



Applications

- String dictionaries
  - With prefix lookup, predecessor, …
  - Exploit prefix compression
- Monotone perfect hash functions
  - “Hollow” or “Blind” tries [ALENEX 09]
  - Binary tree (no need store branching chars)
  - No need to store node labels, just lengths (skips)

Height vs. performance

- Tries can be deep – no guarantee on height
- Bad with pointer-based trees
  - ~1 cache miss per *child* operation
- Worse with succinct tree encodings
  - Need to access several directories
  - *Many* cache misses per *child* operation
  - Large constants hidden in the O(1)

Problem

- Given a sorted set *S* of *K* prefix-free strings
  - $\{s_1, s_2, s_3, \ldots., s_K\}$
  - With total length $N = \sum |s_i|$
  - With each character drawn from $\sum = \{1,2,..,\sigma\}$

- Create a data structure that answers the following queries for a pattern p

  - **Lookup(p)** returns -1 if p is not in S or a unique ID in the interval [K]
  - **Access(*i*)** retrieves the string with ID = *i*

Linearized Trie Lower Bound: LT(S)

- Let T be the compacted trie of S
- Let E be # characters on the edges of T
- Let t be the total number of nodes of T

Theorem. Any succinct encoding of S requires at least

$$\boxed{LT(S) = E \log \sigma + B(t-1,E)}$$

bits in the worst case.

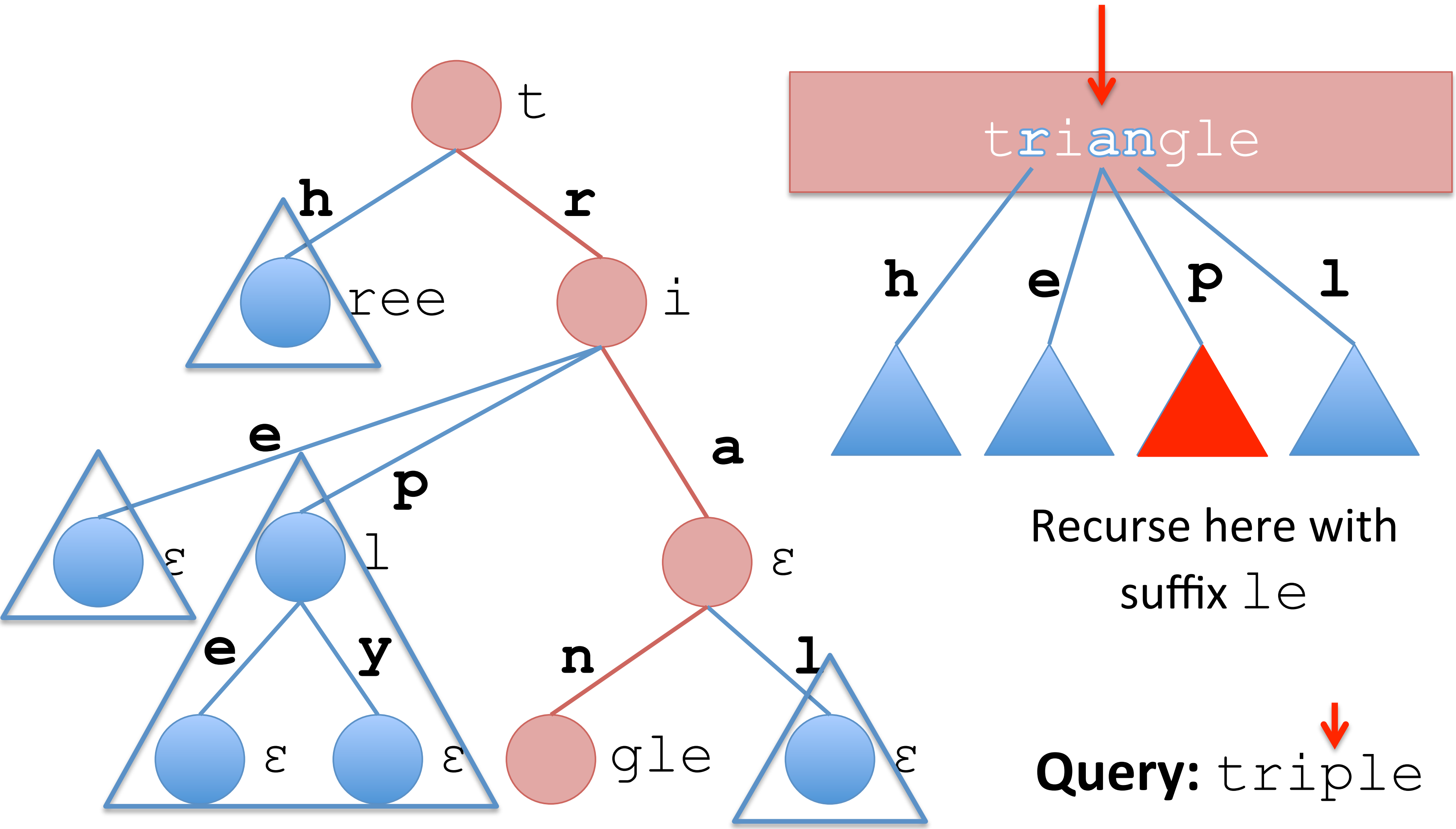
$$B(n,m) = \left\lceil \log \binom{m}{n} \right\rceil$$

Upper Bounds

Encoding for S that

- supports Lookup(p) and Access(i) = p in O(p) time
- uses (1 + o(1)) LT(S) + O(K) bits
- easy to implement and fast

Path decomposition



Recurse here with suffix le

Query: triple

Centroid path decomposition

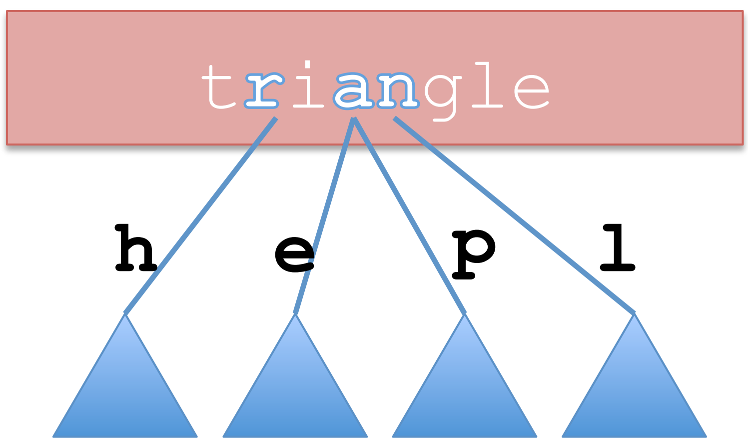
- Decompose along the **heavy paths**
  - choose the edge that has most descendants
- Height of the decomposed tree: O(log n)
  - Usually lower
- Average height

	Web Queries	URLs	Synthetic
Compacted trie	11.0	18.1	504.4
Centroid trie	**5.2**	**6.2**	**2.8**
Hollow trie	50.8	67.3	1005.3
Centroid hollow trie	**8.0**	**9.2**	**2.8**

Succinct encoding

- [PODS 08] presents a succinct data structure for centroid path-decomposed tries
- Not practical: need complex operations on succinct trees
- Simpler and practical encoding
- This encoding enables also simple compression of the labels

Succinct encoding




```
L :  t1ri2a1ngle
BP:   (  (((    )
B :   h  epl
```

(spaces added for clarity)

- Node label written literally, interleaved with number of *other* branching characters at that point in array **L**
- Corresponding branching characters in array **B**
- Tree encoded with DFUDS in bitvector **BP**
  - Variant of Range Min-Max tree [ALENEX 10] to support Find{Close,Open}, more space-efficient (Range Min tree)

Compression of **L**

```
...$...index.html$....html$....html$...index.html$
```

```
...
3 index
...
5 .html
...
```

Dictionary

```
...$...35$...5$...5$...35$
```

- Dictionary codewords shared among labels
- Codewords do not cross label boundaries (`$`)
- Use vbyte to compress the codeword ids

Compression of **L**

- Node labels (`t`<u>**1**</u>`ri`<u>**2**</u>`a`<u>**1**</u>`ngle`, `l`<u>**1**</u>`e`, …):
  - each label is suffix of a string in the set
  - interleaved with few "special characters" <u>**1**</u>, <u>**2**</u>, <u>**3**</u>,…
- Compressible if strings are compressible
- Dictionary and parsing computed with modified Re-Pair
  - Domain-specific compression can be used instead
- Decompression overhead negligible

Experimental results (time)

- Experiments show gains in time comparable to the gains in height
- Confirm that bottleneck is traversal operations

	Web Queries	URLs	Synthetic
Trie	3.5	7.0	119.8
Centroid trie	**2.4**	**4.3**	**5.1**
Hollow trie [ALENEX 09]	16.6	22.4	462.7
Hollow trie	7.2	13.9	137.1
Centroid hollow trie	**2.8**	**4.4**	**11.1**

(microseconds, lower is better)

Code available at `https://github.com/ot/path_decomposed_tries`

Experimental results (space)

- For strings with many common prefixes, even non-compressed trie is space-efficient
- Labels compression considerably increases space-efficiency
- Decompression time overhead: ~10%

	Web Queries	URLs	Synthetic
Hu-Tucker Front Coding	40.9%	24.4%	19.1%
Centroid trie	55.6%	22.4%	17.9%
Centroid trie + compression	**31.5%**	**13.6%**	**0.4%**

(compression ratio, lower is better)

Code available at `https://github.com/ot/path_decomposed_tries`

Part III:

Wavelet Trie for a **Sequence** of Strings

Sequence of strings ≠ Set of strings

- **Time order** e.g. in data analytics:

  What has been the most accessed domain during winter vacation?

- **Binary relation** e.g. in web graphs and social networks:

  How did friendship links change during winter vacation?

Indexed String Sequences

(foo, bar, foobar, foo, bar, bar, foo)

0 1 2 3 4 5 6

- **Queries**
  - **Access(i):** access the *i*-th element
    - Access(2) = `foobar`
  - **Rank(s, pos):** count occurrences of *s* before *pos*
    - Rank(`bar`, 5) = 2
  - **Select(s, i):** find the *i*-th occurrence of a *s*
    - Select(`foo`, 2) = 6

Prefix operations

(foo,	bar,	foobar,	foo,	bar,	bar,	foo)
0	1	2	3	4	5	6

- **Queries**
  - **RankPrefix(p, pos):** count strings prefixed by *p* before *pos*
    - RankPrefix(`foo`, 5) = 3
  - **SelectPrefix(p, i):** find the *i*-th string prefixed by *p*
    - SelectPrefix(`foo`, 2) = 3

Example: storing relations

- Write the **columns** as string sequences
  - Store them separately
  - Reduce relational operations to **sequence queries**

	User			Likes URL
0	Leonard	battl	0	et/wow/
1	Penny	tmz.	1	ı
2	Sheldon	ba	2	et/wow/
3	Penny	thec	3	secakefactory.com
4	Leonard	wi	4	ia.org/Star_Trek
5	Sheldon	wi	5	ia.org/String_theory
6	Sheldon	ma	6	com

What does **Sheldon** like?

Who likes pages from domain **wikipedia.org**?

Other operations: range counting, …

Some notation

```
(foo, bar, foobar, foo, bar, bar, foo)
  0    1     2      3    4    5    6
```

- Sequence S, |S| = n
  - In the example n = 7
- String set S_{set} is *unordered set* of *distinct* strings appearing in S
  - In the example, `{foo, bar, foobar}`, $|S_{set}| = 3$
  - Also called *alphabet* but is LARGE and VARIABLE
- Sequence symbols can also be integers, characters, …
  - As long as they are binarized to strings

Dynamic sequences

We want to support the following operations:

- **Insert(s, pos)**: insert the string *s* immediately before position *pos*
- **Append(s)**: append the string *s* at end of the sequence (special case of **Insert**)
- **Delete(pos)**: delete the string at position *pos*

If data structure only supports **Append**, we call it *append-only*, otherwise *dynamic* (or *fully dynamic*)

All the operations can change S_{set}

The Wavelet Trie

- The Wavelet Trie is a Wavelet Tree on sequences of binary strings ($S_{set} \subset \{0, 1\}^*$)
- Supports Access/Rank(Prefix)/Select(Prefix)
- Fully dynamic…
- … or append only (with better bounds)
- The string set S_{set} *need not be known in advance*

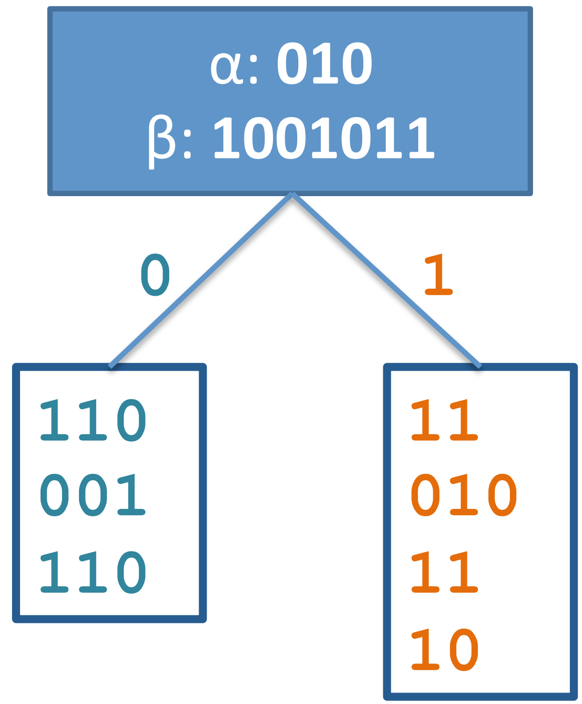
Wavelet Trie: Construction

Sequence of **binary** strings

Branching bit: β

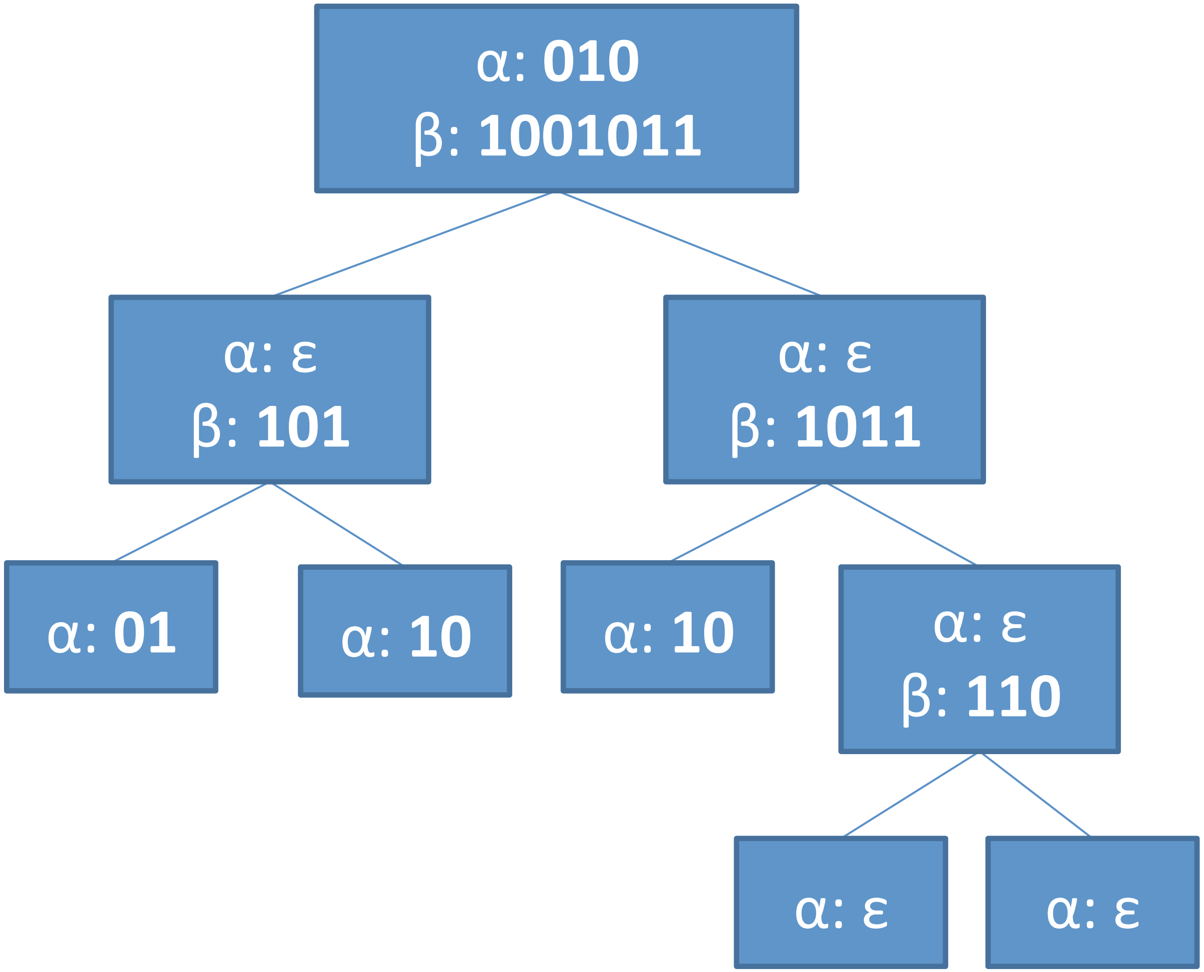
```
0101 11
0100 110
0100 001
0101 010
0100 110
0101 11
0101 10
```

Common prefix: α



Wavelet Trie: Construction

0 1 0 1 1 1
0 1 0 0 1 1 0
0 1 0 0 0 0 1
0 1 0 1 0 1 0
0 1 0 0 1 1 0
0 1 0 1 1 1
0 1 0 1 1 0



Space analysis

- Information-theoretic lower bound

$$LB(S) = LT(S_{set}) + nH_0(S)$$

  - LT is the information-theoretic lower bound for storing a *set* of strings (see previous slides)
  - $nH_0(S)$ is the 0-th order entropy of a sequence S of n atomic items

- $\hat{h}$ = avg height of the Wavele Trie (WT)

Space analysis

- Static WT:

$$LB(S) + o(\hat{h}n)$$

- Append-only WT:

$$LB(S) + PT(S_{set}) + o(\hat{h}n)$$

  – $PT(S_{set})$: space taken by the Patricia Trie

- Fully dynamic WT:

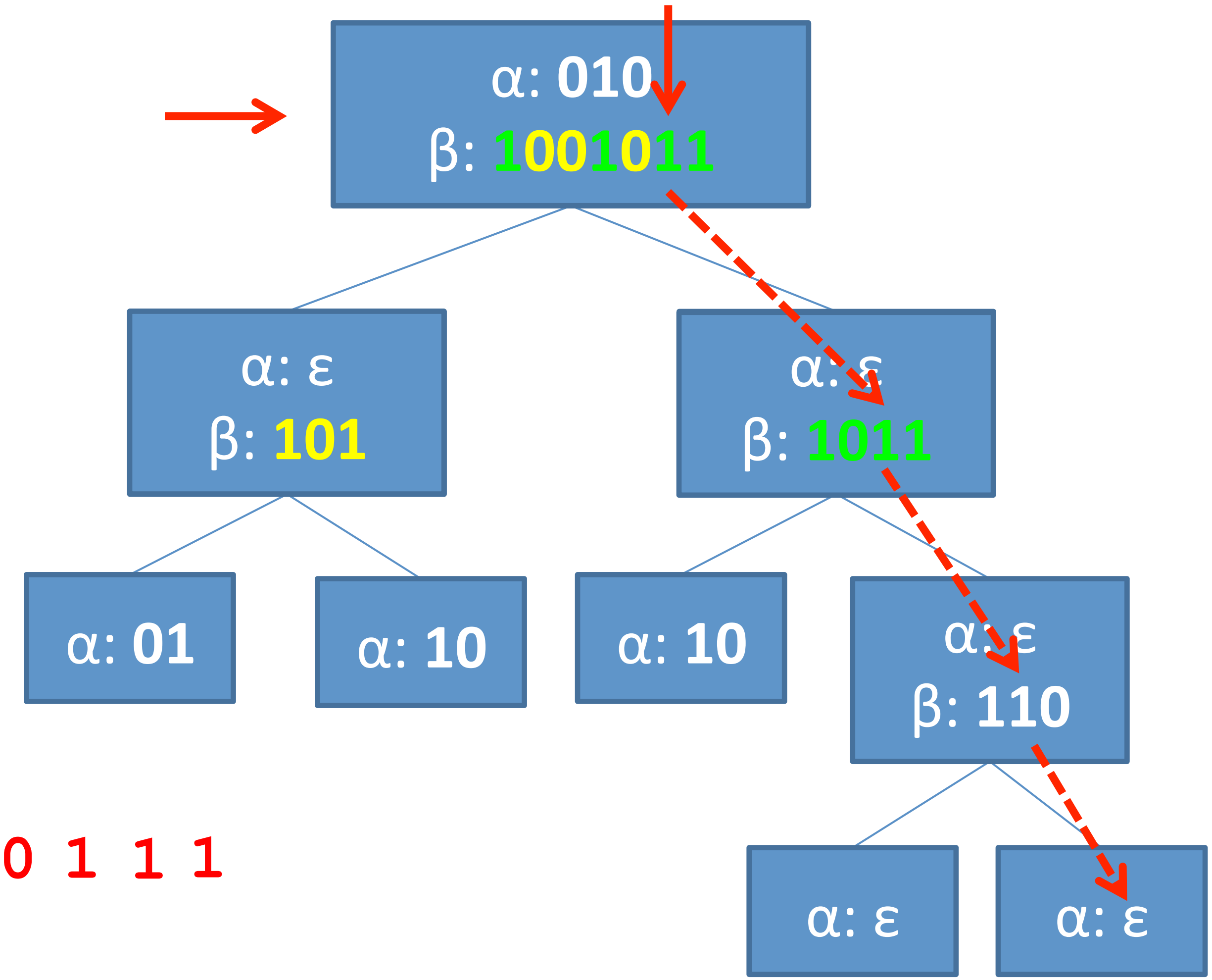
$$LB(S) + PT(S_{set}) + O(nH_0(S))$$

Operations time complexity

- Need new dynamic bitvectors to support *initialization* (create a bitvector $\mathtt{0}^n$ or $\mathtt{1}^n$)
- Static and Append-only Wavelet Trie
  - All supported operations on *s* in $O(|s| + h_s)$
  - h_s is number of nodes traversed by string *s*
- Fully dynamic Wavelet Trie
  - All supported operations in $O(|s| + h_s \log n)$
  - Deletion may take $O(|\hat{s}| + h_s \log n)$ where $\hat{s}$ is longest string in the trie

Wavelet Trie: Access

0 010111
1 0100110
2 0100001
3 0101010
4 0100110
5 010111
6 010110

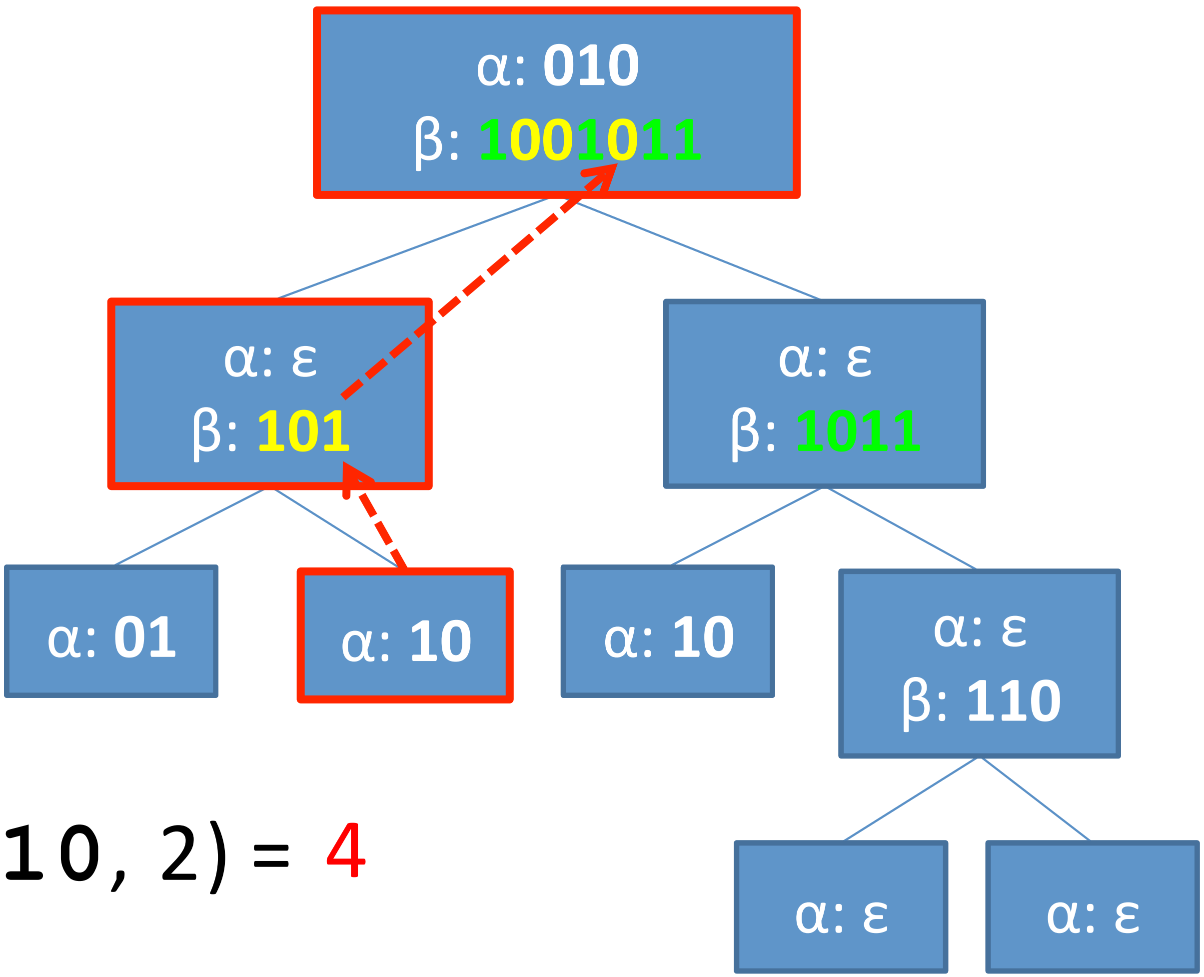


Access(5) = 010 1 1 1

Rank is similar

Wavelet Trie: Select

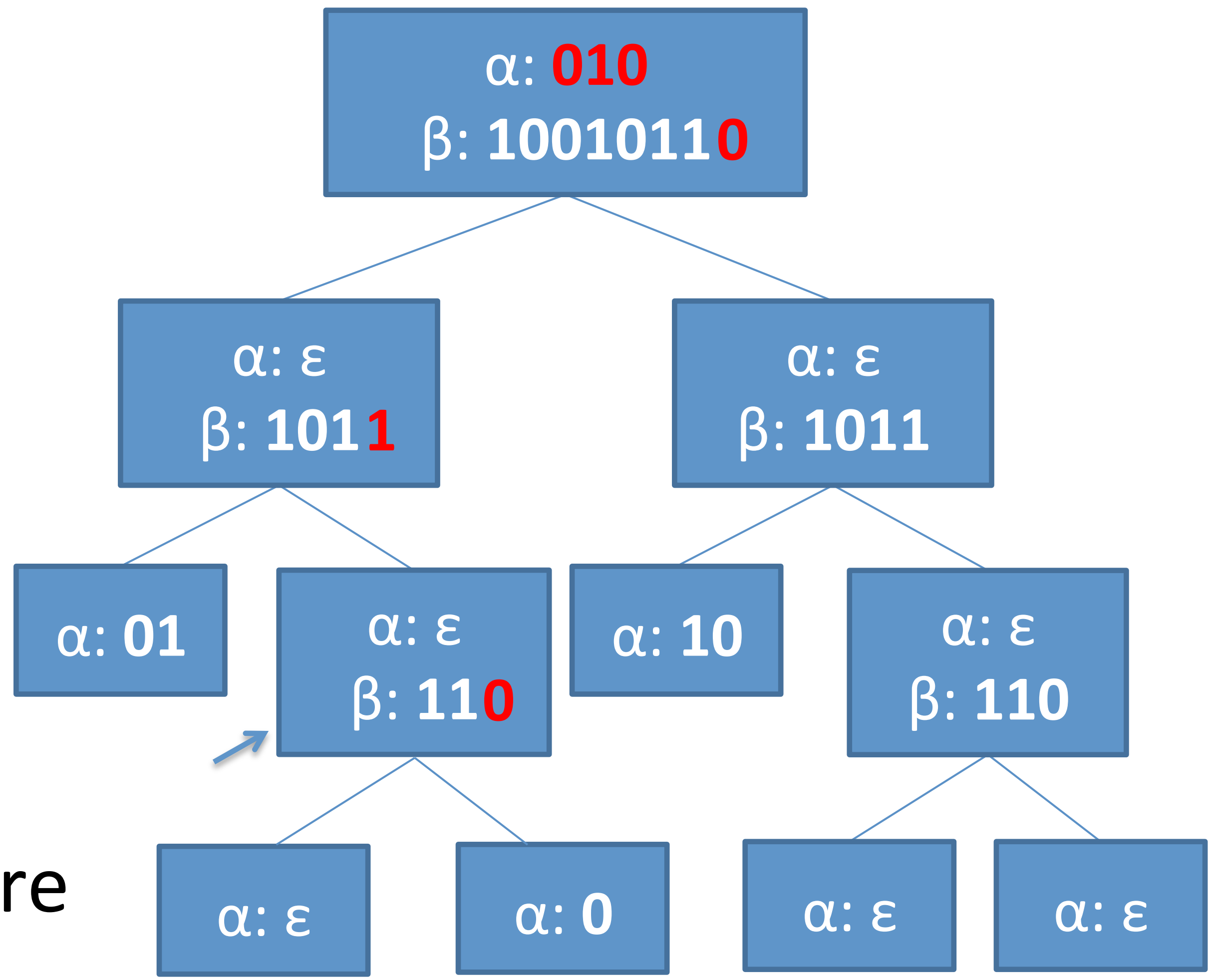
0 010111
1 0100110
2 0100001
3 0101010
4 0100110
5 010111
6 010110



Select(0100110, 2) = 4

Wavelet Trie: Append

010111
0100110
0100001
0101010
0100110
010111
010110
010010



Insert/Delete are similar

Summary

- Compressed suffix arrays for a **string** = **sequence of symbols**
- Wavelet trees for a string
- Succinct tries for a **set of strings**
- Wavelet tries for a **sequence of strings**

- Industrial applications: json processing, query log analysis, auto-completion
- Code being developed is available at https://github.com/ot/succinct

Thanks for your attention!

Questions?