The Complexity of Games and Puzzles

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Why Games and Puzzles?

- Serve as great **teaching** tools.
- Results are more likely to be read, understood and appreciated by **people outside the academia**.
- It is **fun**! ☺

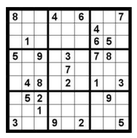
Computational Complexity

Popular puzzles and games are usually hard (not interesting if finding a solution or designing a strategy is straightforward).

Puzzles are NP-Complete:

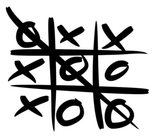
- It is easy to describe and verify the solution of the puzzle;
- It is hard to come up with the solution.





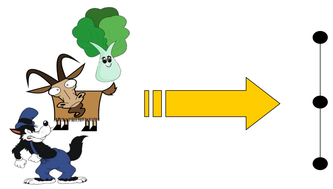
Games are** (at least) **PSPACE-Hard:

- Impossible to verify quickly an optimal strategy for one of the players;
- Efficient to describe any intermediate configuration of a game.

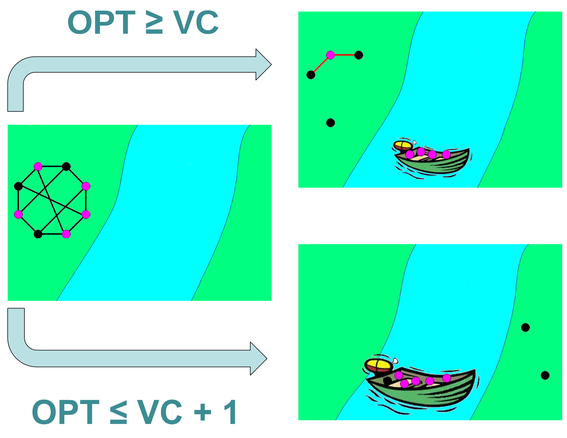


Wolf-Goat-Cabbage

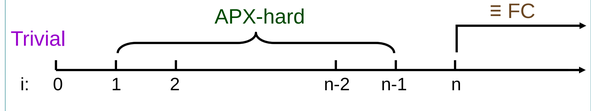
Generalization:



Reduction from Vertex Cover:



Trip Constraint Version:



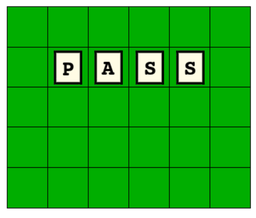
(Work that appears in Lampis & M (FUN '07 and Csobra & Woeginger ESA '09)

Scrabble

Our model

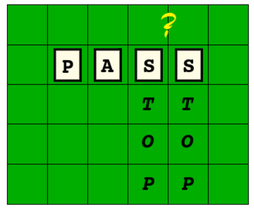
- Generalized version (unbounded board);
- Deterministic, full-information version;
- Made-up language;
- No special squares on the board;
- All tiles worth the same amount of points.

Hardness due to formation:



STOP or SPOT?

Hardness due to placement:

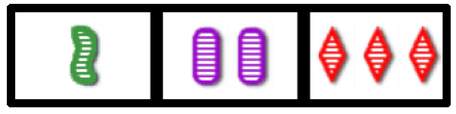


Both tasks are <u>independently</u> **PSPACE-hard.**

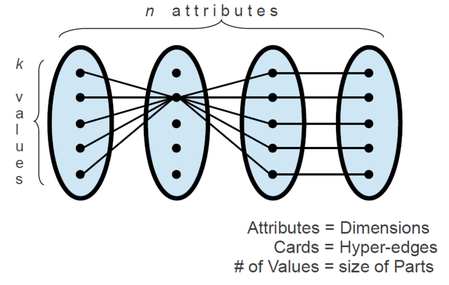
Set Game

Rules:
Each card has 4 attributes and each attribute has 3 values.

- Deal 12 cards
- Find a ***set****:* 3 cards with values for each attribute being either *all the same* or *all different.*



Connection with r-Dim Matching:



Results:

- **Find one set:**
  - **For *k* unbounded:**
- *n* = 2 → P (find a star or a matching)
- *n* ≥ 2 → NP-Complete (*Chaudhuri et al.*)
  - **For *n* unbounded:**
- *k* = 2 → trivial
- *k* parameter → W-hard
- **Find max number of disjoint sets:**
  - For k=3 the problem is NP-hard.