二分決定グラフによる量子回路設計

立命館大・情報理工・山下茂

- 量子回路-2^nx2^nの複素行列
- ある種の量子回路の場合２分決定グラフで効率的に表現可能
- そして，ほぼストレートフォワードに量子回路設計可能
- 解きたい問題

(古典なら2^nの0 / 1)

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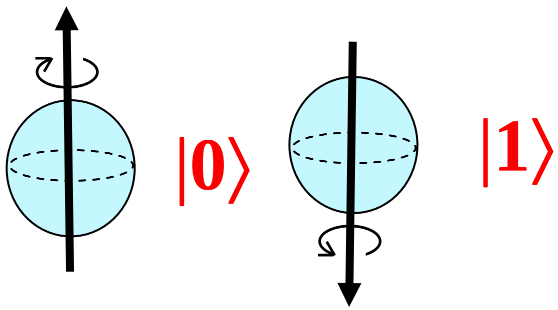
What is Q.C. in short?

To perform computation by using "quantum states," controlling by quantum gates

Quantum States – 2 level physical system

- **light phase** (0° and 90°)
- **spins of electron** (↑ and ↓)
- **energy level of quantum dots (ground and exited)**
- **NMR(average spin of $6x10^{23}$ molecules)**



Quantum Gate

We can control quantum states by adding magnetic fields, irradiate an electromagnetic waves, ...

Quantum Bit

$$\sqrt{0.7}\,|0\rangle + \sqrt{0.3}\,|1\rangle$$

Any superposition of two distinguishable quantum states

Quantum Bit & Quantum Gate

Quantum Gate = Certain Physical Operation

$$|0\rangle \;—\boxed{H}—\; \boxed{1/\sqrt{2}\,|0\rangle + 1/\sqrt{2}\,|1\rangle}$$

Superposition of $|0\rangle$ and $|1\rangle$ w.p. ½ each

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad H = 1/\sqrt{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$H\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1/\sqrt{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1/\sqrt{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- **Quantum Bit: 1x2 column vector**
- **Quantum Gate: 2x2 Matrix**

Operations in Q. C.

$$R(\theta)=\begin{pmatrix}\cos\frac{\theta}{2} & -i\sin\frac{\theta}{2}\\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix}\qquad NOT=iR(\pi)=i\begin{pmatrix}\cos\frac{\pi}{2} & -i\sin\frac{\pi}{2}\\ -i\sin\frac{\pi}{2} & \cos\frac{\pi}{2}\end{pmatrix}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

Classical Operation : NOT

$$|0\rangle \underset{NOT}{\Leftrightarrow} |1\rangle \qquad \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}=\begin{pmatrix}0\\ 1\end{pmatrix} \qquad \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}=\begin{pmatrix}0\\ 1\end{pmatrix}$$

- **Quantum Bit: 1x2 column vector**
- **Quantum Gate: 2x2 Matrix**

Quantum Operation

$$R(\theta)=\begin{pmatrix}\cos\frac{\theta}{2} & -i\sin\frac{\theta}{2}\\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix}$$

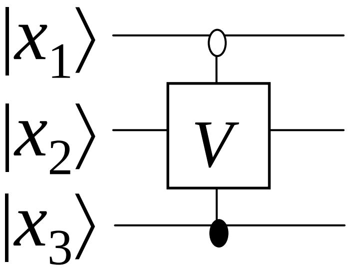
$$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}=\begin{pmatrix}0\\ 1\end{pmatrix}$$

$$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}=\begin{pmatrix}0\\ 1\end{pmatrix}$$

Representation for General Q. C.

- **Quantum Bit: 1x2 column vector**
- **Quantum Gate: 2x2 Matrix**

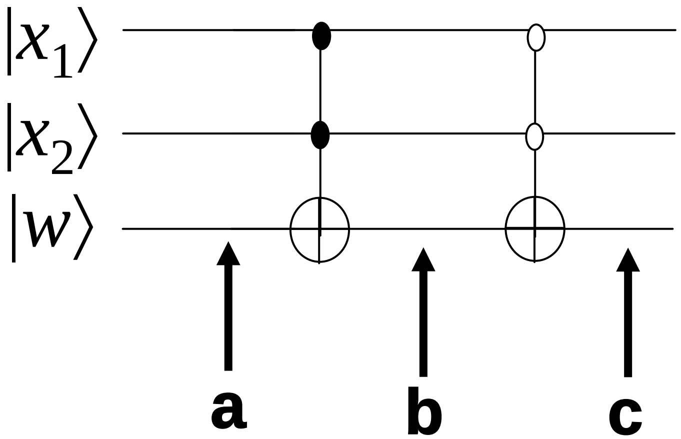
- **n Quantum Bits: 1×2^n column vector**
- **n Quantum Gate: $2^n \times 2^n$ Matrix**



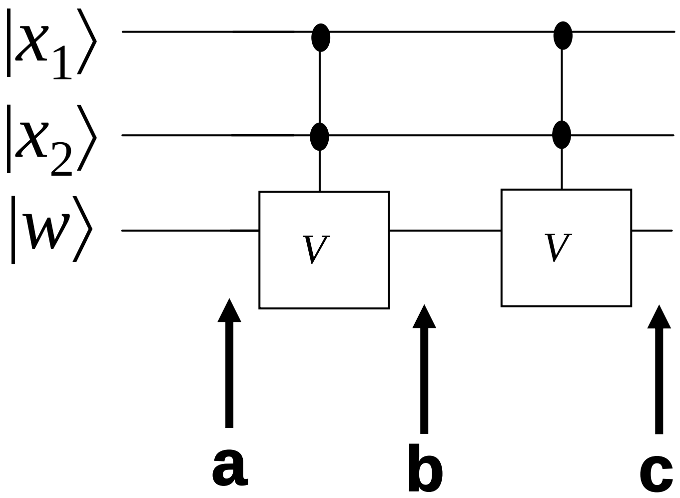
$$V = \frac{1+i}{2}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

SCQCs and Matrix Functions



	a	b	c	f
00	I	I	N	1
01	I	I	I	0
10	I	I	I	0
11	I	N	N	1

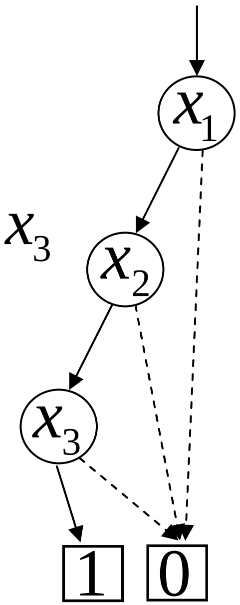


	a	b	c	f
00	I	I	I	0
01	I	I	I	0
10	I	I	I	0
11	I	V	N	1

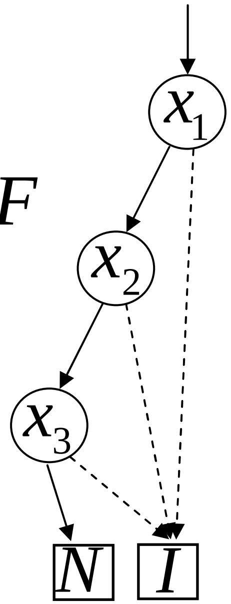
DDMF: Decision Diagram for Matrix Function

$$f = x_1 x_2 x_3$$

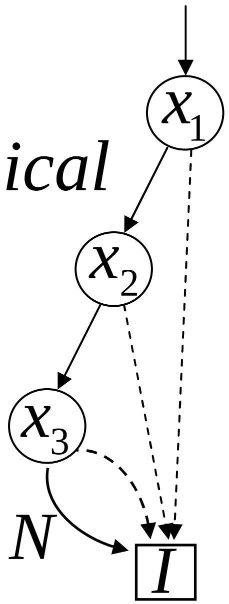
BDD



DDMF

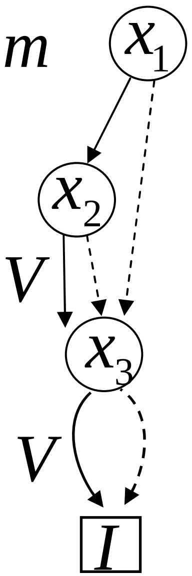


Cannonical



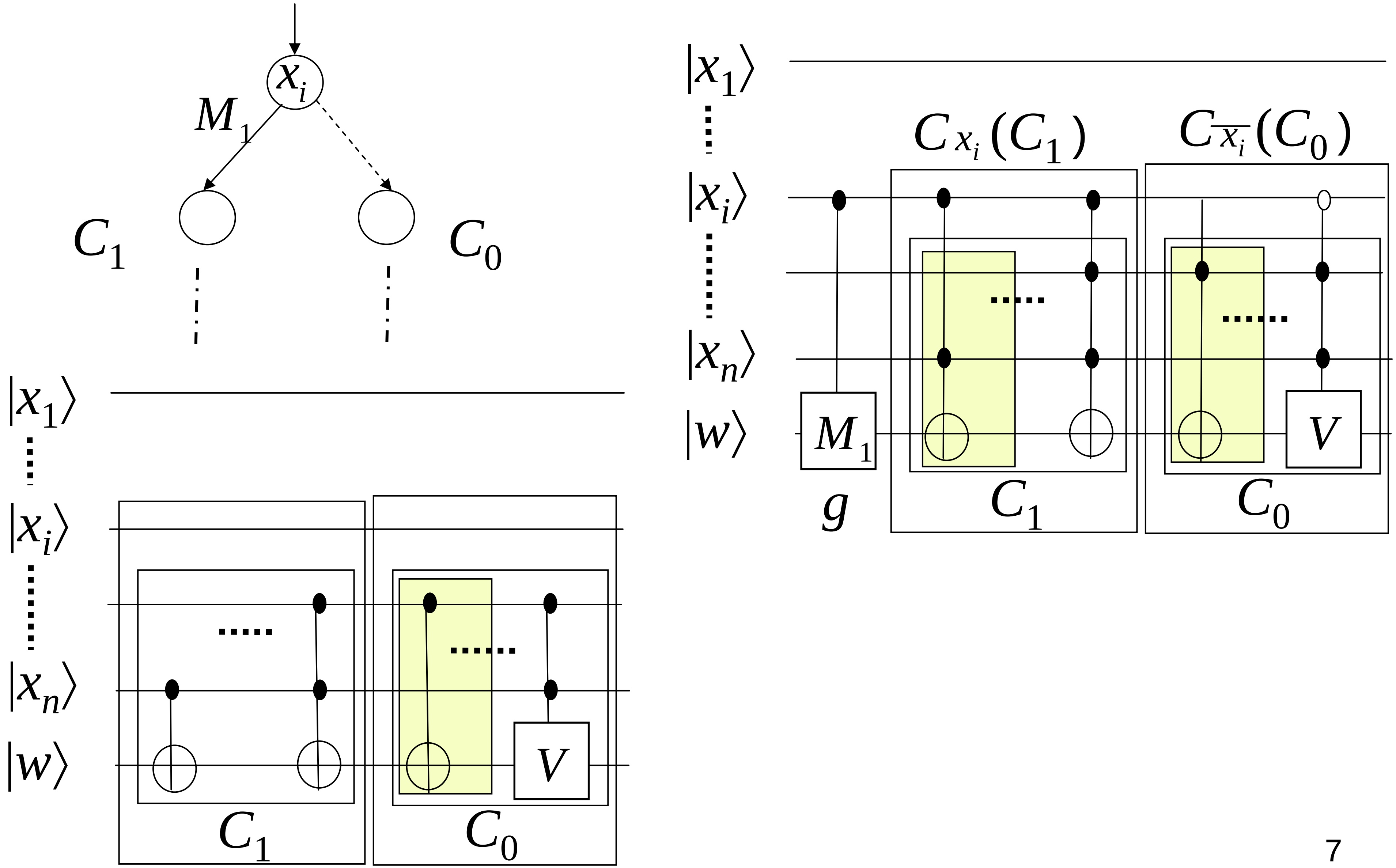
	f	MF	MF_m
000	0	I	I
001	0	I	V
010	0	I	I
011	0	I	V
100	0	I	I
101	0	I	V
110	0	I	V
111	1	N	N

m



DDMF-> Quantum Circuit

Easy to Perform Recursive Algorithm



Problem Formulation

1. **How about variable ordering?**
2. **How to divide the target func. into several MFs?**

$x_1x_2x_3$	X_1	X_2	X_3	R	Target Func.
000	I	I	I	I	I
001	I	I	V	V	N
010	I	V	I	V	N
011	I	V	V	I	N
100	V^{-1}	I	I	V	I
101	V^{-1}	I	V	I	I
110	V^{-1}	V	I	I	I
111	V^{-1}	V	V	V	N

Easy (X_1, X_2, X_3) **?** (R)