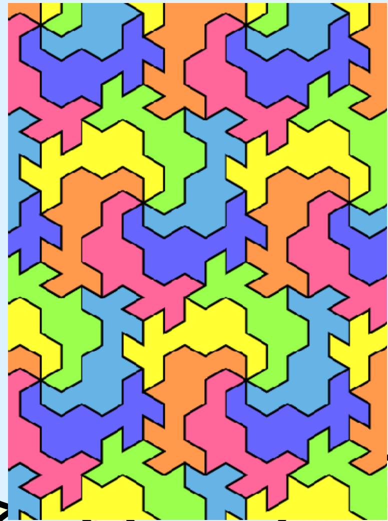
正四面体と他のプラトン立体の間の共通の（辺）展開図に関する研究

上原隆平(JAIST) with
堀山貴史 (埼玉大学)
白川俊博 (ジー・モード)

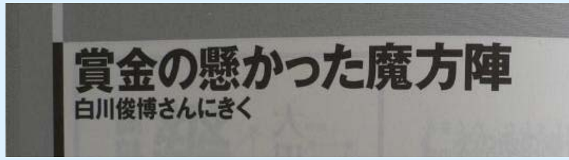
ERATO合宿 2010/11/29-12/1

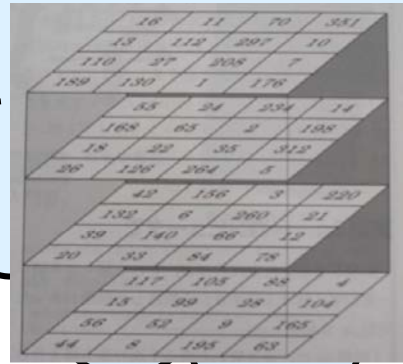
Folding & Unfolding

Tiling

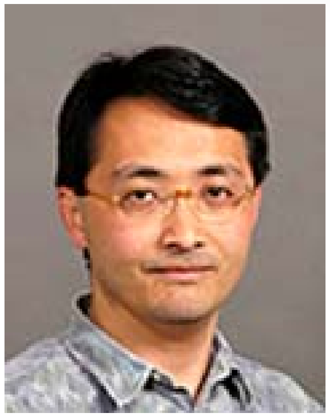


Puzzle





展開



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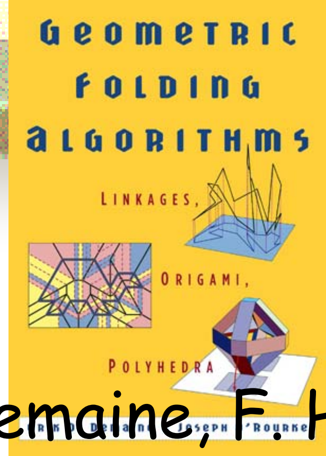
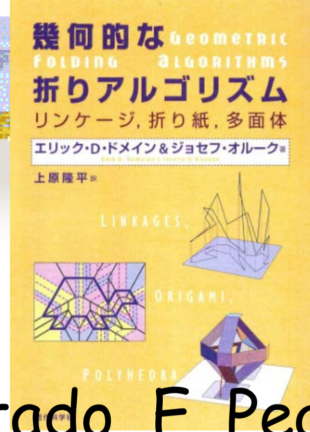
Introduction



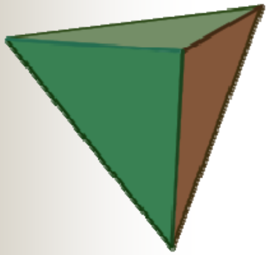


- Open problem 25.6

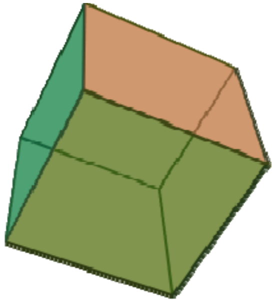
  (by M. Demaine, F. Hurtado, E. Pegg)

  - Can any **Platonic solid** be cut open and unfolded to a polygon that may be refolded to a **different Platonic solid**?

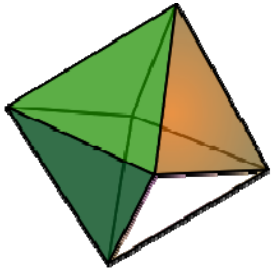
    For ex., may a cube be so dissected to a tetrahedron ?



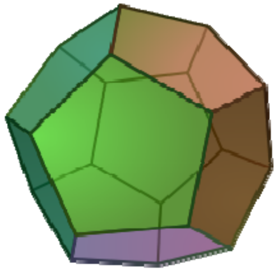
Tetrahedron



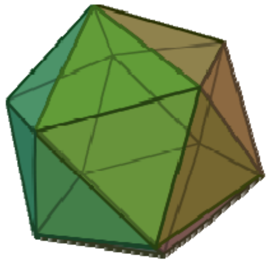
Cube (Hexahedron)



Octahedron

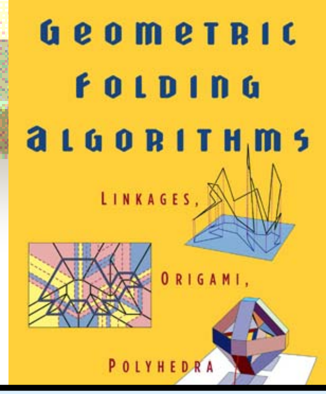


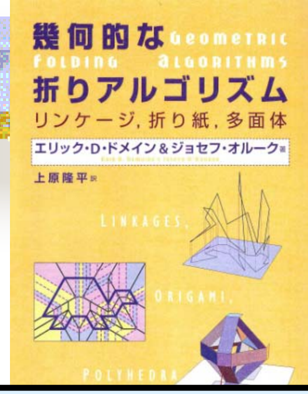
Dodecahedron



Icosahedron

Introduction

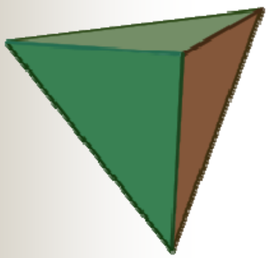


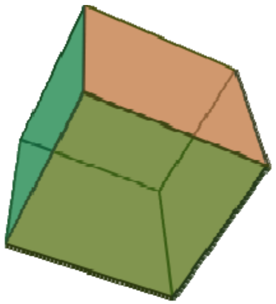


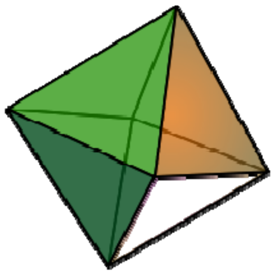
- Open problem 25.6

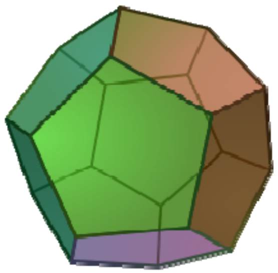
Are there polygons that can be folded to two (or more) different Platonic solids ?

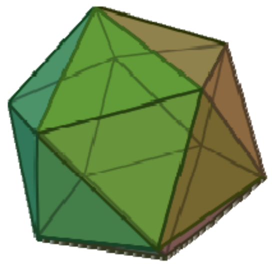
For ex., may a cube be so dissected to a tetrahedron ?





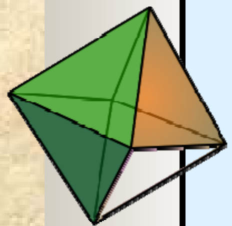




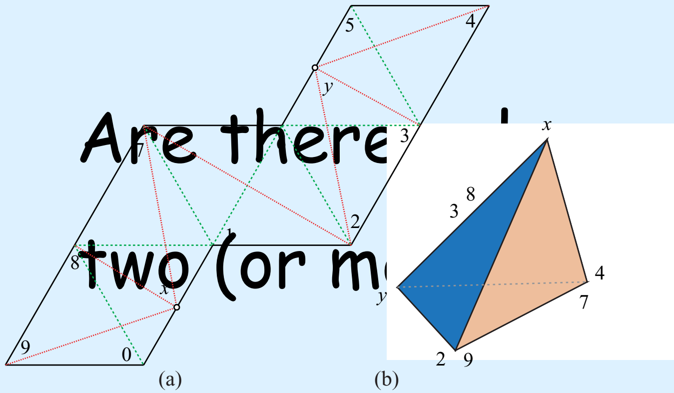


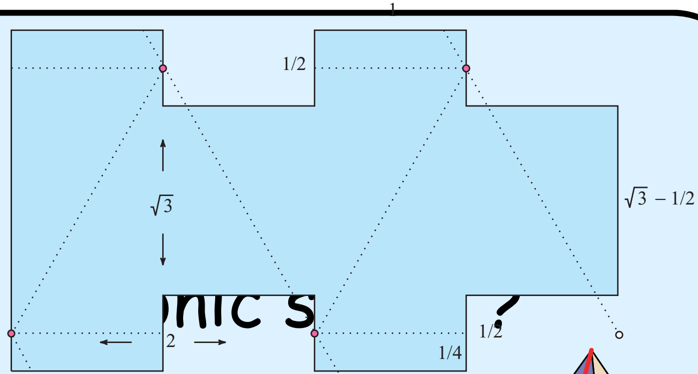
Tetrahedron | Cube (Hexahedron) | Octahedron | Dodecahedron | Icosahedron

Introduction



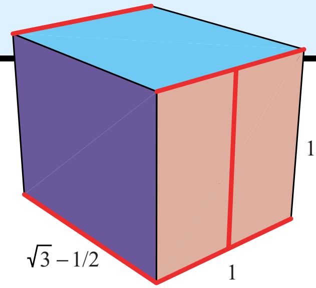
Are there … that
two (or m… erent …nic s… ?

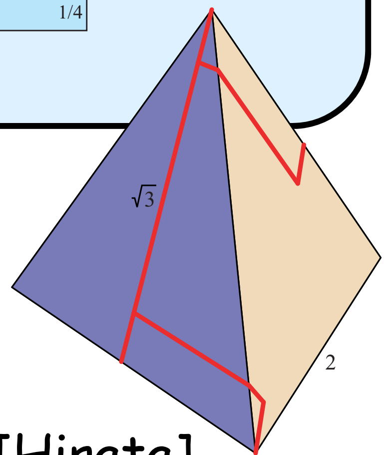




Close call ! [O'Rourke]

Regular Octahedron
⇔ Tetramonohedron
(tetrahedron all of whose
faces are congruent)



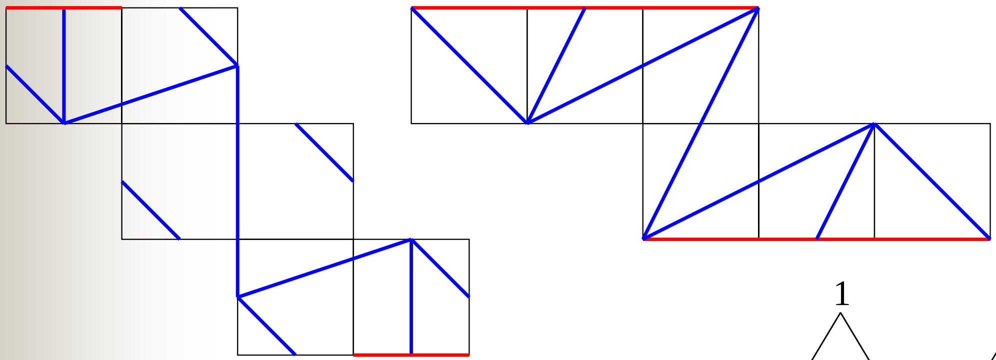


Close call !! [Hirata]

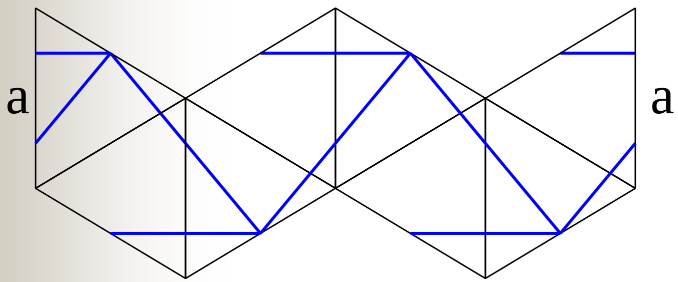
Regular Tetrahedron
⇔ Box 1 x 1 x 1.232

Are there polygons that can be folded to two (or more) different Platonic solids ?

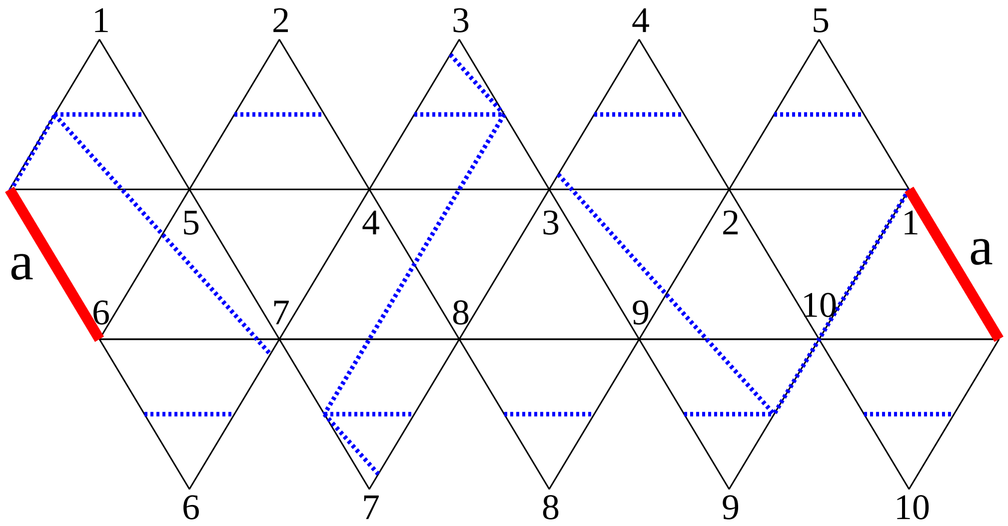
Close calls



Cube (Regular hexahedron)
⇔ Tetramonohedron
1 : 0.972 : 0.972



Regular Octahedron
⇔ Tetramonohedron
1.0072 : 0.9965 : 0.9965



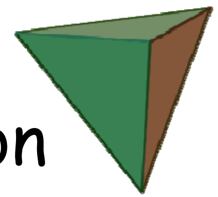
Regular Icosahedron
⇔ Tetramonohedron 1 : 1.145 : 1.25

Are there polygons that can be folded to two (or more) different Platonic solids ?

Our Results

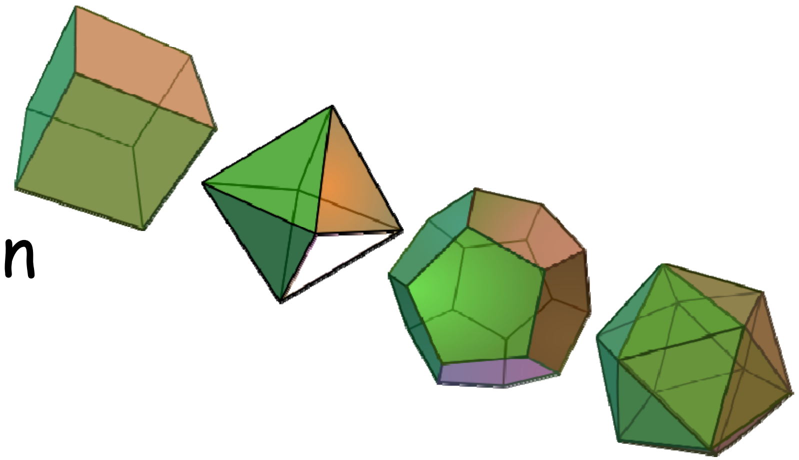
There exists no polygon **P** s.t.

(1) **P** is a **development** of a regular tetrahedron

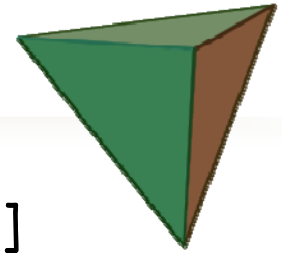


(2) **P** is a **development by edge-cutting** of

- a regular hexahedron
- a regular octahedron
- a regular dodecahedron
- a regular icosahedron



The Key Theorem

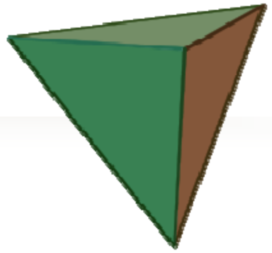


- Theorem [Akiyama 2007, Akiyama Nara 2007]

  Let **P** be a development of a **regular tetrahedron**.

  Then, **P** is a **tiling**. That is, **P** **fills a plane**.

The Key Theorem



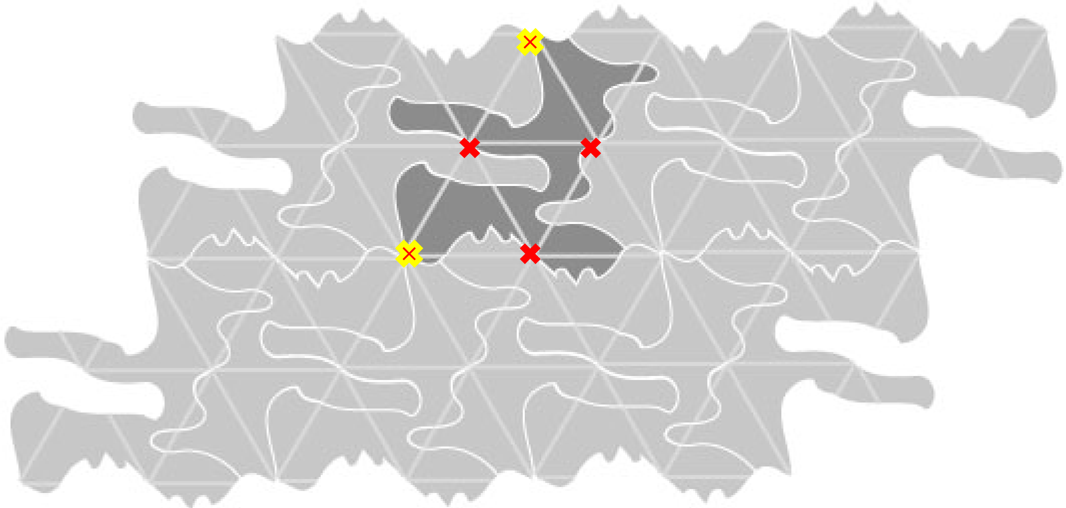
- Theorem

  P is a development of a **regular tetrahedron** iff

  (1) **P** has a **p2 tiling**, i.e., tiling by 180° rotations

  (2) **4** of the **rotation centers** define the triangular lattice

  (3) no two of the 4 rotation centers belong to the same equivalent class on the tiling

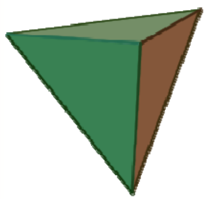


Conclusion

Open problem:

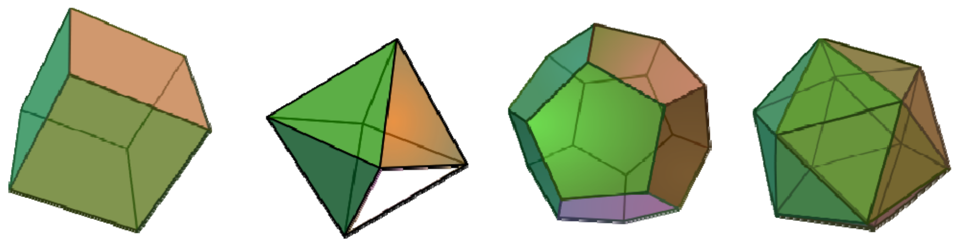
Are there polygons that can be folded to two (or more) different Platonic solids ?

Development of



vs

Development of



Future Work: 1. by edge-cutting

2. Relation among these 4